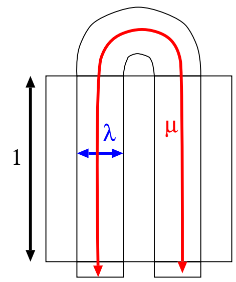
Homework #9
Nonlinear dynamics and chaos

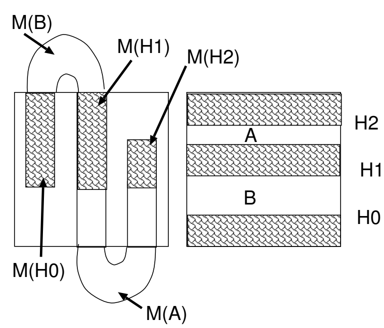
1. Horseshoe map:

   (a) Calculate the fractal dimension of the invariant set (intersect of horizontal and vertical stripes) of the horseshoe map with stretching and contraction factor μ, λ. See figure.



   (b) For stretching and contraction factors both equal to 3, plot the set $H_{ijk} \cap V_{ijk}$. Mark the squares containing the fixed points, and the location of the points belonging to the period-3 orbit $\overline{011}$. To what accuracy can you find them?

   (c) Limited shift Horseshoe (Ott, 1st ed, problem 1, p 148): Consider the horseshoe-type map shown in the following figure:



   The map is equivalent to a shift operation over the three symbols 0, 1 and 2. Note, however, that region H_2 is mapped only to H_1, while H_0 and H_1 are mapped only to H_1 and H_2. This means that in a symbolic representation of the dynamics using bi-infinite symbols, 2 must be followed by 1, and 0 and 1 may be followed by either 1 or 2 (did I get this right…?). That is, the shift operation is limited to certain combinations of the symbols. Write all the possible periodic orbits up and including period 4. You may want to read the 1 page discussion of such "limited shift" maps in Ott (pp 113-114 in 1st ed).

2. (Vered Rom-Kedar) Consider the Henon map:

$$x_{n+1} = a + by_n - x_n^2 \quad (1)$$
$$y_{n+1} = x_n \quad (2)$$

   (a) Find its fixed points and their stability (as a function of the parameters a, b).

(b) Draw, in the parameter space ((a,b) plane) the bifurcation curves of this map (recall that for maps bifurcations occur when eigenvalues cross the unit circle). In particular, denote the saddle-node bifurcation curve by $a_0(b)$

(c) Use the program Henon.m to check your results.

(d) Let R be the larger root of:

$$\rho^2 - (|b|+1)\rho - a = 0.$$

Denote by S the square centered at the origin with vertices $\pm R, \pm R$ Examine the image of the square S under the Henon map. Show that there exists a curve $a_2(b)$ such that for $a > a_2(b)$ the image of S under the Henon map creates a horseshoe map.

(e) Plot the Henon attractor and examine its structure.

(f) Plot the stable and unstable manifolds of the Henon map using a method based on their definition given in class.

(g) **Extra credit:** Prove that the stretching and contraction conditions occur for $a > a_2(b)$ so that the dynamics of the Henon map in this regime on S is the same as that of the horseshoe map.