EPS 131 Problem set 6 SOLUTIONS

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1. and 2: Answers in back of Open University text.

Question 4.3 (a) The component of the Earth's rotation about a vertical axis at latitude ϕ is $\Omega \sin \phi$. Vorticity is $2 \times$ angular velocity, and so the planetary vorticity possessed by a parcel of water on the surface of the Earth at latitude ϕ is $2\Omega \sin \phi$. This is the expression for the Coriolis parameter, f (Section 3.1.2.).

(b) (i) and (ii) At the North and South Poles, the planetary vorticity will be equal to $2\Omega \sin 90°$, i.e. 2Ω, because $\sin 90° = 1$. As illustrated in Figure 4.7(a), the surface of the Earth rotates anticlockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere. If we use the convention, given in the text, that clockwise rotation corresponds to negative vorticity, the planetary vorticity possessed by a parcel of fluid at the South Pole will be written as -2Ω.

(iii) At the Equator, $\phi = 0$, and so planetary vorticity $= 2\Omega \sin 0 = 0$.

Question 4.4 (a) (i) A body of water carried southwards from the Equator is moving into regions of increasingly negative planetary vorticity (Figure 4.7(a)). (ii) Its absolute vorticity $f + \zeta$ must remain constant, so to compensate for the decrease in f, its relative vorticity ζ must *increase*. In other words, the water must increasingly acquire a tendency to rotate with positive vorticity (i.e. anticlockwise) in relation to the Earth. (If the force originally driving the water ceases to act, we will have a simple inertia current, anticlockwise in the Southern Hemisphere – Figure 3.7(a).)

(b) Negative relative vorticity will be acquired by a body of water from (i) winds blowing in a clockwise direction, *and* (ii) from cyclonic winds in the Southern Hemisphere, because these are also clockwise (Figure 4.5).

4.6 on next page.

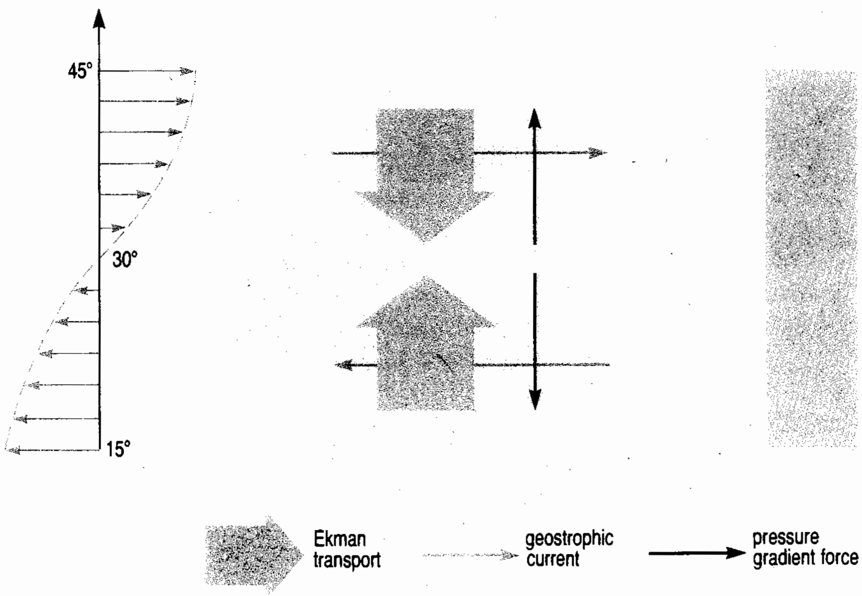
Question 4.7 The flow pattern in (2) is symmetrical, like that in (1), and so the existence of the Coriolis force *per se* cannot be the reason for the intensification of western boundary currents. However, when the Coriolis force is made to increase with latitude – albeit linearly – the flow pattern does show the asymmetry observed in the real oceans. Figure 4.12 therefore strongly suggests that the intensification of western boundary currents is a result of the fact that the Coriolis parameter (and hence the Coriolis force) increases with latitude.

2. cont'd

Question 4.6 (a) The resemblance is fairly close. In both cases the overall motion is clockwise (cf. Figure 4.5): the part of the wind field between 15° and 30° N corresponds to the more southerly part of the subtropical anticyclonic gyre, especially the Trade Winds, which have a strong easterly component; the part between 30° and 45° N corresponds to the westerlies forming the higher-latitude part of the subtropical anticyclones.

(b) Ekman transport is to the right of the wind in the Northern Hemisphere and so will be in the directions shown by the wide arrows in Figure A5.

(c) The Ekman transports towards the centre of the ocean will converge and lead to a raised sea-surface (cf. Figure 3.24(c) and (d)). As a result, there will be horizontal pressure gradient forces acting northwards and southwards away from the centre (black arrows on Figure A5), so that geostrophic currents will flow in the directions shown by the thin blue arrows (cf. Figure 3.24).



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3. $$0 = f(u_x + v_y) + \beta v + \frac{1}{\rho}\frac{\partial}{\partial z}(\tau_y^{(x)} - \tau_x^{(y)}) - r(u_y - v_x)$$ $[\beta \equiv \frac{\partial f}{\partial y}]$ Knauss (6.27)

$$0 = -f w_z + \beta v - \frac{1}{\rho}\frac{\partial}{\partial z}\,\mathrm{curl}\,\tau + r\zeta\,, \quad [\zeta \equiv v_x - u_y\,,\ u_x + v_y + w_z = 0]$$

Assuming $\tau = \tau_{wind}$ at the surface and $\tau \approx 0$ below,

integrate the above equation $\int_{-H=-5km}^{-50m} dz\{\}$ (region where $\tau \approx 0$)

using $w(-50m) = \frac{1}{\rho_0 f}\,\mathrm{curl}\,\tau$ (Ekman pumping) to get

$$\boxed{\underbrace{-\frac{1}{\rho_0 H}\,\mathrm{curl}\,\tau_{wind}}_{\text{wind stress}} + \underbrace{\beta v}_{\text{planetary vorticity}} + \underbrace{r\zeta}_{\text{friction}} = 0}$$