Homework #3

Introduction to physical oceanography

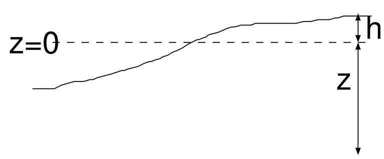
1. Calculating the volume transport of water by the Gulf Stream and the Antarctic circumpolar current in Sverdrup (1Sv=million meter cubed per second):

   (a) find and plot maps of the sea surface height in the area of the Gulf Stream and the Drake Passage. (One place to look is under the home page of Michael A. Chupa, `http://www.erc.msstate.edu/~chupa/`, see the picture `http://www.erc.msstate.edu/~chupa/istv/global16.jpg`; this image is also under the "supporting material" link on the course home page; it is a model result rather than data, but appropriate for our purpose).

   (b) What is the sea surface height difference across the Gulf Stream and Antarctic circumpolar current (in the Southern Ocean)?

   (c) Ignoring density variations, show that the pressure in the ocean is given by

$$p(x,z) = g\rho_o(h(x) - z)$$



   where $h(x)$ is the surface height as function of the east-west coordinate across the Gulf Stream, ρ_0 is the constant water density, g is gravity, and z is the depth. Note that for the circumpolar current, the pressure and surface height are a function of the north-south direction, y, instead.

   (d) Use one of the two geostrophic equations

$$fv = \frac{1}{\rho_0}\frac{\partial p}{\partial x} \quad (1)$$

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \quad (2)$$

   to calculate how much water is transported by the Gulf Stream and the Antarctic circumpolar current per second (express your results in Sverdrup). Hint: the transport is given by $\Delta X \int_{z=0}^{-H} v dz = \Delta X H v$ where ΔX is the width of the Gulf Stream/ circumpolar current, H is the maximum depth to which the current extends and assuming that the velocity is independent of depth within this range. Use $H = 2km$ for the Gulf Stream and the bottom depth for the Antarctic circumpolar current.

2. Follow the argument in Knauss and explain why the term $\vec{\Omega} \times \vec{\Omega} \times \vec{\mathbf{x}}$ is negligible relative to the Coriolis acceleration $2\vec{\Omega} \times \vec{\mathbf{u}}$.

3. **An optional challenge problem:**

(a) Calculate the equilibrium shape of the surface of water in a bucket rotating about its axis with angular velocity Ω if it contains water of uniform density.

(b) Interpret the results in terms of the forces acting on the water.

Hint: The water is assumed to rotate with the bucket, so that it is at rest ($u = 0$) in the appropriate rotating frame. Use the equations derived in class to calculate the pressure, $P(x,y,z)$, and then use the boundary condition $P = 0$ at the surface to calculate the shape of the surface as $z = h(x,y)$.