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Introduction to Physical Oceanography
Ekman Pumping

Assume that we have easterly trades winds between the latitudes 15°N and 30°N and westerlies between the latitudes 30°N and 45°N. The domain is defined by $-L \leq y \leq L$ where $L = 1670km$.

The wind stress is given by

$$\tau_x = \tau_0 sin\left(\frac{\pi y}{2L}\right) \quad (1)$$
$$\tau_y = 0 \quad (2)$$

where $\tau_0 = 0.15N/m^2$ is the maximum wind stress.

From the expression above, the wind stress is negative and decreasing in magnitude from 15°N to 30°N. Being zero at 30°N, it changes sign and starts increasing northward. In the northern hemisphere, the net transport in the Ekman layer is to the right of the wind stress and his magnitude depends on the amplitude of the wind stress, we therefore expect to have convergence of water at every latitude with a maximum at 30°N (see figure 1). When we have convergence of the water at the surface, we expect downwelling to occur ($w_E < 0$).

1. Calculate the Ekman pumping at 30°N assuming that $\rho_0 = 1028kg/m^3$ and $f = 2\Omega sin(30°N)$.

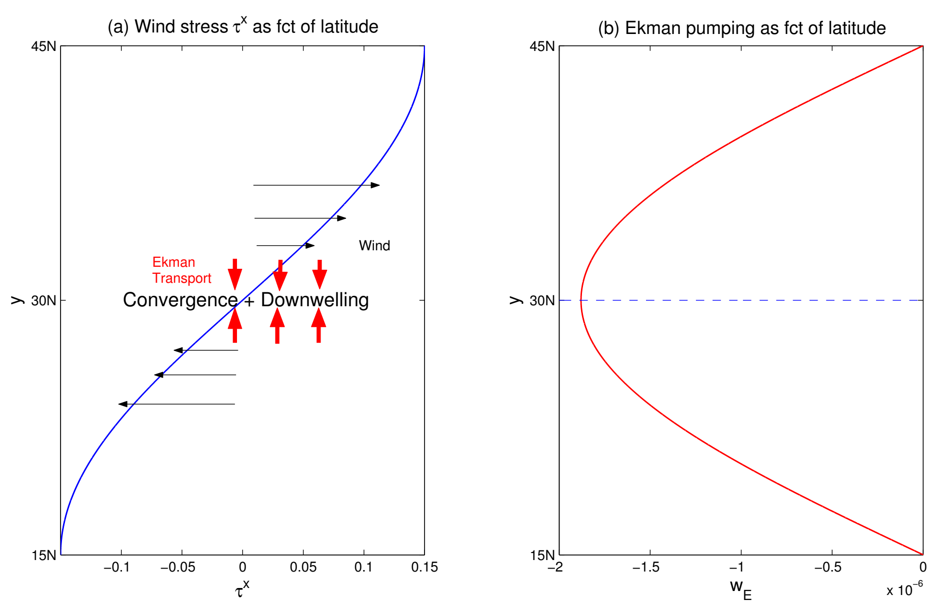

Figure 1: (a) Wind stress and water transport, (b) Ekman velocity w_E.

$$w_E = \frac{1}{\rho_0 f}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right) \quad (3)$$

$$
\begin{aligned}
w_E &= \frac{1}{\rho_0 f}\left[\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial y}\left(\tau_0 sin\left(\frac{\pi y}{2L}\right)\right)\right] && (4)\\
w_E &= \frac{-\tau_0 \pi}{2L\rho_0 f} cos\left(\frac{\pi y}{2L}\right) && (5)
\end{aligned}
$$

In our domain $-L \leq y \leq L$, $cos\left(\frac{\pi y}{2L}\right)$ is always positive such that $w_E \leq 0$ as expected (see figure 1b).

2. Find the vertical volume flux W over the entire $15°N - 45°N$ strip of the North Pacific (assume the width to be equal to $\Delta x = 8700km$).

$$
\begin{aligned}
W &= \int_A w_E dA = \Delta x \int_{-L}^{L} w_E dy\\
W &= \frac{-\Delta x \tau_0}{\rho_0 f} \int_{-L}^{L} \frac{\partial}{\partial y}\left(sin\left(\frac{\pi y}{2L}\right)\right)\\
W &= \frac{-\Delta x \tau_0}{\rho_0 f} sin\left(\frac{\pi y}{2L}\right)\Big|_{-L}^{L}\\
W &= \frac{-2\Delta x \tau_0}{\rho_0 f} \approx -3.48 \times 10^7 m^3/s \approx -35Sv
\end{aligned}
$$

Again, as expected the volume transport is negative, meaning that we have downwelling of water.