Notes for climate dynamics course (EPS 231)

Eli Tziperman

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Contents

1 Introduction

This is an evolving detailed syllabus of EPS 231, see course web page, all course materials are available under the downloads directory. If accessing from outside campus or via the university wireless network, you will need to connect via the Harvard VPN.

Homework assignments (every 9-10 days) are 50% of final grade, and a final course project constitutes the remaining 50%. We strongly encourage you to discuss and work on homework problems and collaborate on solving them with other students (and with the teaching staff, of course), but you must write your final answers in your own words. You should ensure that any written work you submit for evaluation reflects your own work and your own understanding of the topic. There is an option to take this course as a pass/fail with approval of instructor.

2 Basics, energy balance, multiple climate equilibria

Downloads available here.

2.1 Multiple equilibria, climate stability, greenhouse

1. `energy_balance_0d.pdf` with the graphical solutions of the steady state solution to the equation $CT_t = (Q/4)(1-\alpha(T)) - \varepsilon\sigma T^4$ obtained using energy_balance_0d.m, and then the quicktime animation of the bifurcation behavior.

2. Some nonlinear dynamics background: saddle node bifurcation ((p 45, Strogatz, 1994) or Applied Math 203 notes p 47), and then the energy balance model as two back to back saddle nodes and the resulting hysteresis as the insolation is varied;

3. Climate implications: (1) faint young sun paradox! (2) snowball (snowball obs from Ed Boyle's lecture);

4. 2-level greenhouse model (notes) and slide on actual mechanism (ppt).

2.2 Small ice cap instability

(Time permitting)

1. Introduction: The Budyko and Sellers 1d models Simple diffusive energy balance models produce an abrupt disappearance of polar ice as the global climate gradually warms, and a corresponding hysteresis. The SICI eliminates polar ice caps smaller than a critical size (18 deg from pole) determined by heat diffusion and radiative damping parameters. Below this size the ice cap is incapable of determining its own climate which then becomes dominated, instead, by heat transport from surrounding regions. (Above wording from Winton (2006), teaching notes below based on North et al. (1981)).

2. Derive the 1d energy balance model for the diffusive and Budyko versions (eqns 22 and 33): Assume the ice cap extends where the temperature is less than -10C, the ice-free areas have an absorption (one minus albedo) of $a(x) = a_f$, and in the ice-covered areas $a(x) = a_i$; approximate the latitudinal structure of the annual mean insolation as function of latitude using $S(x) = 1 + S_2P_2(x)$, with $P_2(x) = (3x^2 - 1)/3$ being the second Legendre polynomial (HW); add the transport term represented by diffusion $(r^2\cos\theta)^{-1}\partial/\partial\theta(D\cos\theta\partial T/\partial\theta)$, which, using $x = \sin\theta$, $1 - x^2 = \cos^2\theta$ and $d/dx = (1/\cos\theta)d/d\theta$ gives the result in the final steady state 1D equation (22),

$$-\frac{d}{dx}D(1-x^2)\frac{dT(x)}{dx} + A + BT(x) = QS(x)a(x,x_0).$$

   as boundary condition, use the symmetry condition that $dT/dx = 0$ at the equator, leading to only the even Legendre polynomials.

3. (time permitting) Alternatively to the above, we could model the transport term a-la Budyko as $\gamma[T(x) - T_0]$ with T_0 being the temperature averaged over all latitudes, and the temperature then can be solved analytically (HW).

4. How steady state solution is calculated: eqns 22-29; then 15 and 37, the equation just before 37 and the 4 lines in the paragraph before these equations. Result is Fig. 8: plot of Q (solar intensity) as function of x_s (edge of ice cap). Analysis of results: see highlighted Fig. 8 in sources directory, unstable small ice cap (which cannot sustain its own climate against heat diffusion from mid-latitudes), unstable very-large ice cap (which is too efficient at creating its own cold global climate and grows to a snowball), and stable mid-size cap (where we are now).

5. Heuristic explanation of SICI: Compare Figs 6 and 8 in North et al. (1981), SICI appears only when diffusion is present. It is therefore due to the above mentioned mechanism: competition between diffusion and radiation. To find the scale of the small cap: it survives as long as the radiative effect dominates diffusion: $BT \sim DT/L^2$, implying that $L \sim \sqrt{D/B}$. Units: $[D]$ =watts/(degree K$\times$m^2) (page 96), $[B]$ =m^2/sec (p 93), so that $[L]$ is non dimensional. That's fine because it is in units of sine of latitude. Size comes out around 20 degrees from pole, roughly size of present-day sea ice!

6. The important lesson(!): p 95 in North et al. (1981), left column second paragraph, apologizing for the (correct...) prediction of a snowball state.

7. This is a complex PDE (infinite number of degrees of freedom), displaying a simple bifurcation structure. In such case we are guaranteed by the central manifold and normal form theorems that it can be transformed to the normal form of a saddle node near the appropriate place in parameter space. First, transformed to center manifold and get an equation independent of stable and unstable manifolds (first page of lecture_04_cntr_mnfld.pdf); next, transform to normal form within center manifold (lecture_03_bif1d2.pdf, p 89).

8. (time permitting) Numerically calculated hysteresis in 1D Budyko and Sellers models: figures ebm1d-budyko.jpg and ebm1d-sellers.jpg obtained using ebm_1dm.m).

9. (time permitting) One noteworthy difference between Budyko and Sellers is the transient behavior, with Budyko damping all scales at the same rate, and Sellers being scale-selective. (original references are Budyko, 1969; Sellers, 1969).

3 ENSO

Downloads available here.

3.1 ENSO background and delay oscillator models

Sources: Woods Hole (WH) notes (Cessi et al., 2001), lectures 0, 1, 2, here, plus the following: Gill's atmospheric model solution from Dijkstra (2000) technical box 7.2 p 347; recharge oscillator from Jin (1997) (section 2, possibly also section 3);

- The climatological background: easterlies, walker circulation, warm pool and cold tongue, thermocline slope (ppt, and lecture 1 from WH notes).

- Dynamical basics (WH lecture 1): reduced gravity equations on an equatorial beta plane (note error in derivation of $\nabla_H p_2$ in notes). Equatorial Rossby and Kelvin waves, thermocline slope, SST dynamics, atmospheric heating and wind response to SST from Gill's model. The coupled ocean-atmosphere feedback.

- The heuristic delayed oscillator equation from section 2.1 in WH notes. One detail to note regarding how do we transition from $+\hat{b}h_{\text{off-eq}}(t-[\frac{1}{2}\tau_R+\tau_K])$ to $-\bar{b}\tau_{eq}(t-[\frac{1}{2}\tau_R+\tau_K])$ and then to $-bT(t-[\frac{1}{2}\tau_R+\tau_K])$: $h_{\text{off-eq}}$ depends on the Ekman pumping off the equator. In the northern hemisphere, if the wind curl is positive, the Ekman pumping is positive, upward ($w_E = \text{curl}(\vec{\tau}/f)/\rho$), and the induced thermocline depth anomaly is therefore negative (a shallowing signal). The wind curl may be approximated in terms of the equatorial wind only (larger than the off-equatorial wind), consider the northern hemisphere:

$$h_{\text{off-eq}} \propto -w_{\text{off-eq}}^{Ekman} \propto -\text{curl}(\tau_{\text{off-eq}}) \approx \partial_y \tau_{\text{off-eq}}^{(x)} \approx (\tau_{\text{off-eq}}^{(x)} - \tau_{\text{eq}}^{(x)})/L \propto -\tau_{\text{eq}}^{(x)}.$$

Finally, as the east Pacific temperature is increasing, the wind anomaly in the central Pacific is westerly (positive), leading to the minus sign in front of the T term,

$$-\tau_{\text{eq}}^{(x)}(t-[\frac{1}{2}\tau_R+\tau_K])/L \propto -T(t-[\frac{1}{2}\tau_R+\tau_K]).$$

- Next, the linearized stability analysis of the Schopf-Suarez delayed oscillator from the WH notes section 2.1.1. The dispersion relation in the WH notes is $\sigma = 1-3\bar{T}^2-\alpha\exp(-\sigma\delta)$. Its real part is $0=\sigma_R-(1-3\bar{T}^2-\alpha\exp(-\sigma_R\delta)\cos(-\sigma_I\delta))$, and its imaginary part is $0=\sigma_I-(-\alpha\exp(-\sigma_R\delta)\sin(-\sigma_I\delta))$. The following image shows the roots for an example with two unstable roots and many stable ones. Show time series of numerical solution of this model for values on both sides of the first bifurcation (damped and self-sustained) point, obtained using delay_Schopf_Suarez_1989.m.

- Self-sustained vs damped: nonlinear damping term and, more importantly, the proximity of ENSO to the first bifurcation point beyond steady state, discuss Hopf bifurcation from non linear dynamics notes pp 26-28 here or Strogatz (1994).

- (time permitting) A more quantitative derivation of delay oscillator, starting from the shallow water equations and using Jin's two-strip approximation (WH notes, section 2.2).

3.2 ENSO's irregularity

3.2.1 Chaos

- ENSO phase locking to seasonal cycle: flows on a circle, synchronization/ phase locking, Huygens clocks, firefly and flashlight example from Strogatz. Connection to ENSO and the seasonal cycle.

- ENSO irregularity as chaos driven by the seasonal cycle: circle map and quasi-periodicity route to chaos:

$$\theta_{n+1}=f(\theta_n)=\theta_n+\Omega-\frac{K}{2\pi}\sin 2\pi\theta_n, \quad \theta_n = mod(1)$$

  $K=0$ and quasi-periodicity, $0<K<1$ and phase locking, Arnold tongues; $K=1$ and the devil's staircase. $K>1$, overlapping of resonances and chaos. Transition from circle map to pendulum and to delayed oscillator driven by seasonal cycle. (time permitting): winding number: $\lim_{n\to\infty}(f^n(\theta_0)-\theta_0)$, not taking θ_n as mod(1) for this calculation, Farey tree

- (time permitting) Some generalities on identifying quasi-periodicity route to chaos in a complex system, including delay coordinate phase space reconstruction.

- References for phase locking: Strogatz (1994), for quasi-periodicity route to chaos: Schuster (1989). For both: course notes for applied math 203, pages 71,75,77-83 in lecture_bif1d2_eli.pdf. delay coordinate phase space reconstruction in lecture_bif2d3_eli.pdf.

- (time permitting) Slides on transition to chaos in CZ model.

3.2.2 Noise

- Non normal amplification (similar to WH notes, but maximizing $\Psi(\tau)^T\Psi(\tau)$ instead of $|d\Psi(t=0)/dt|^2$ as described in WH notes.): transient growth; geometric view using a 2×2 example of $d\mathbf{x}/dt = A\mathbf{x}$ where $\mathbf{x} = (x,y)$, with eigenvectors on unit circle, derivation of optimal i.c that maximize $\mathbf{x}(\tau)^T\mathbf{x}(\tau)$ as first eigenvector of B^TB where B is the propagator $B = \exp(A\tau)$; corresponding eigenvalue is the amplification factor from the initial conditions to the amplified state.

- WWBs in observations: seem stochastic, seen to precede each El Nino event, affect Pacific by forcing of equatorial Kelvin waves. Show Hovmoller diagram with WWBs and SST from Yu et al. (2003), in jpg file; wind stress sequence showing WWB evolution from Vecchi and Harrison (1997) (Figs on pages 41, 42, 43 at the end of this report); effects of wind bursts on SST and thermocline depth (heat content) from Mcphaden and Yu (1999) Figs 1,2,3 (last one is model results); ocean-only model response to a strong WWB: Zhang and Rothstein (1998), Figs. 4, 5, showing the response to a wind burst after 10 days and after several months;

- (time permitting) Stochastic optimals: (can also leave this to be discussed in the context of THC) the derivation from Tziperman and Ioannou (2002): consider a stochastically forced linear system:
$$\dot{P} = AP + f(t)$$
solution is
$$P(\tau) = e^{A\tau}P(0) + \int_0^\tau ds\, e^{A(\tau-s)} f(s) = B(\tau,0)P(0) + \int_0^\tau ds\, B(\tau,s) f(s)$$
variance of the solution is given by
$$\begin{aligned} var(\|P\|) &= \langle P_i(\tau)P_i(\tau)\rangle - \langle P_i(\tau)\rangle\langle P_i(\tau)\rangle \\ &= \left\langle \int_0^\tau ds \int_0^\tau dt B_{il}(\tau,s) f_l(s) B_{in}(\tau,t) f_n(t) \right\rangle \\ &= \int_0^\tau ds \int_0^\tau dt B_{il}(\tau,s) B_{in}(\tau,t) \langle f_l(s) f_n(t)\rangle \end{aligned}$$

  Specifying the noise statistics as separable in space and time, with C_{ln} being the noise spatial correlation matrix and $D(t-s)$ the temporal correlation function (delta function for white noise),

$$\langle f_l(s) f_n(t) \rangle = C_{ln} D(t-s)$$

we have

$$\begin{aligned} var(\|P\|) &= \int_0^\tau ds \int_0^\tau dt B_{il}(\tau,s) B_{in}(\tau,t) C_{ln} D(t-s) \\ &= Tr(\int_0^\tau ds \int_0^\tau dt [B^T(\tau,s) B(\tau,t)] C) \\ &\equiv Tr(ZC) \end{aligned}$$

This implies that the most efficient way to excite the variance is to make the noise spatial structure be the first eigenvector of Z. To show this, show that eigenvectors of Z maximize $J = Tr(CZ) = Z_{ij} C_{ji}$; assuming that the spatial noise structure is f_i, we have $C_{ij} = f_i f_j$ and we need to maximize $Z_{ik} f_j f_k + \lambda(1 - f_k f_k)$; differentiating wrt f_i we get that f_i is an eigenvector of Z.

- Do WWBs look like stochastic optimals? Show stochastic optimals in different models and discuss their model dependence, using Figs. 11, 12 and 17 in Moore and Kleeman (2001).

- Are WWBs actually stochastic, or are their statistics a strong function of the SST (Fig. 2, Tziperman and Yu, 2007), making the stochastic element less relevant?

3.3 Teleconnections

- Motivation for ENSO teleconnection: show pdf copy of impacts page from El Nino theme page.

- Further motivation: results of barotropic model runs from Hoskins and Karoly (1981) Figures 3,4,6,8,9, showing global propagation of waves due to tropical disturbances.

- General ray tracing theory based on notes-ray-tracing.pdf. Note that this is a non-rigorous derivation, not using multiple-scale analysis.

- Derivation of dispersion relation Rossby wave in presence of a mean zonal flow. In Cartesian coordinates, start from potential vorticity conservation, $(\partial_t + u\partial_x + v\partial_y)(v_x - u_y + \beta y) = 0$; let $u = U(y) + u'$, $v = v'$, and linearize, to find $(\partial_t + U\partial_x)(v'_x - u'_y) + (\beta - U_{yy})v' = 0$; introduce stream function $v' = \psi_x$, $u' = -\psi_y$ and effective beta $\beta_{\text{eff}} = \beta - U_{yy}$ to find $(\partial_t + U\partial_x)\nabla^2\psi + \beta_{\text{eff}}\psi_x = 0$; paper uses the same equation, in spherical coordinates, Mercator projection.

- Qualitative discussion based on dispersion relation, from Hoskins and Karoly (1981) after solution 5.23. Start from dispersion relation for stationary waves, $0 = \omega = \bar{u}_M k - \frac{\beta_M k}{k^2+l^2}$, and define $K_s = (\beta_M/u_M)^{1/2} = k^2 + l^2$. Based on these only, discuss trapping by jet. Note that if $k > K_s$ then l must be imaginary. Therefore, as the wave propagates northward from the low latitude, with a constant k while K_s gets smaller due to changes in mean flow and effective beta (Fig. 13a,b), this implies evanescent behavior in latitude past a critical latitude, and trapping of the ray at the critical latitude. However, see Jeff Shaman's description for why the ray is reflected rather than being trapped at the critical latitude due to prognostic dl/dt equation allowing $dl/dt < 0$ when $l = 0$, leading to the ray turning back south.

- (time permitting) For baroclinic atmospheric waves, $0 = \omega = \Omega(k,l,y) = \bar{u}_M k - \frac{\beta_M k}{k^2+l^2+L_R^{-2}}$, so that $K_s = (\beta_M/u_M)^{1/2} - L_R^{-2} = k^2 + l^2$, and they are more easily trapped, and are going to be trapped at the equator with a scale of the Rossby radius of deformation which is some 1000km or so (see discussion on page 1195 left column).

- Derivation of ray tracing equations from the beginning of subsection 5b, equations 5.1-5.16. Because the mean flow and therefore the dispersion relation are x and t-independent, we have k =constant and ω =constant along a ray, while $d_g l/dt \equiv (\partial_t + \mathbf{c}_g \cdot \nabla)l = -\Omega_y$ and $d\mathbf{x}/dt = \mathbf{c}_g$.

- To get some idea of the amplitude of the propagating waves, assume the meridional wavenumber varies slowly in latitude, $l = l(\varepsilon y)$, corresponding to a medium that varies slowly, on a scale longer than the wavelength. Use the WKB solution (Bender and Orszag (1978) section 10.1): start with equation 5.18. Substituting into 5.9 this leads to $d^2P/dy^2 + l^2(\varepsilon y)P = 0$ for $l^2(\varepsilon y)$ defined in 5.20; to transform to standard WKB form, define $Y = \varepsilon y$ so that $\varepsilon^2 d^2P/dY^2 + l^2(Y)P = 0$; try a WKB solution corresponding to a wave-like exponential with a rapidly varying phase plus a slower correction $P = \exp(S_0(Y)/\delta + S_1(Y))$ to find

$$\varepsilon^2[(S_0'/\delta + S_1')^2 + (S_0''/\delta + S_1'')]P + l^2 P = 0;$$

  let $\delta = \varepsilon$ and then $O(1)$ equation is ${S_0'}^2 + l^2(Y) = 0$ so that $S_0 = i\int l(Y)\,dY$ (if l^2 is nearly constant, this simply reduces to the usual wave solution e^{ily}). Next, consider $O(\varepsilon)$ equation which, after using the $O(1)$ equation, is $2S_0'S_1' + S_0'' = 0$ and the solution is $S_1' = -S_0''/(2S_0') = -(dl/dY)/(2l) = -d/dY(\ln l^{1/2})$ so that $S_1 = \ln l^{-1/2}$ which means that the wave amplitude is $l^{-1/2}$. This gives the solution in Hoskins and Karoly (1981) equation (5.21, 5.23), see further discussion there.

- Show rays for constant angular momentum flow, $\bar{u}_M \equiv U/\cos\phi = \bar{\omega}a$, $\beta_M = (2\cos^2\phi)(\Omega + \bar{\omega})/a$ (section 5c, page 1192, Fig. 12) and then the one using realistic zonal flows (Figs. 13, 14, 15, etc);

- Finally, mention that later works showed that stationary linear barotropic Rossby waves excite nonlinear eddy effects which may eventually dominate the teleconnection effects.

4 Thermohaline circulation

Downloads available here.

4.1 Phenomenology, Stommel box model

- Background, schematics of THC, animations of CFCs in ocean, sections and profiles of T, S from here, meridional mass and heat transport, climate relevance; RAPID measurements (Cunningham et al., 2007); anticipated response during global warming; MOC vs THC.
- Stommel model: mixed boundary conditions, Stommel-Taylor model notes; Qualitative discussion on proximity of present day THC to a stability threshold (Tziperman, 1997; Toggweiler et al., 1996). And again the surprising ability of simple models to predict/ explain GCM results (Fig. 2 from Rahmstorf, 1995). See also (Dijkstra, 2000, section 3.1.1, 3.1.2, 3.1.3).

4.2 Scaling and energetics

1. (Time permitting) Scaling for the amplitude and depth of the THC from Vallis (2006) chapter 15, section 15.1, showing that the THC amplitude is a function of the vertical mixing which in turn is due to turbulence. Mention only briefly the issue of the "no turbulence theorem" and importance of mechanical forcing (details in the next section, not to be covered explicitly).
2. (Time permitting) Energetics, Sandstrom theorem stating that "heating must occur, on average, at a lower level than the cooling, in order that a steady circulation may be maintained against the regarding effects of friction" (eqn 15.23 Vallis, 2006). The "no turbulence theorem" in the absence of mechanical forcing by wind and tides (eqn 15.27) without mechanical mixing, and (15.30) with; hence the importance of mechanical energy/ mechanical forcing for the maintenance of turbulence and of the THC. Sections 15.2, 15.3 (much of this material is originally from Paparella and Young, 2002);
3. (Time permitting) Tidal energy as a source for mixing energy (Munk and Wunsch, 1998, Figures 4,5).

4.3 Convective oscillations

- Advective feedback and convective feedback from sections 6.2.1, 6.2.2 in Dijkstra (2000), both show why we expect temperature and salinity to play independent roles.
- (time permitting:) multiple convective equilibria from Lenderink and Haarsma (1994), hysteresis from Fig. 8 in this paper, and "potentially convective" regions in their GCM from Fig. 11.
- Flip-flop oscillations (Welander1982_flip_flop.m) and loop oscillations (Dijkstra, 2000, sections 6.2.3, 6.2.4); (The Lenderink and Haarsma (1994) model, when put in the regime without any steady states, shows exactly the same flip-flop oscillations, yet without the artificial convection threshold necessary in Welander's model).

- Analysis of relaxation oscillations following Strogatz (1994) example 7.5.1 pages 212-213, or nonlinear dynamics course notes: slow phase and fast phases.

- Winton's deep decoupling oscillations may be covered below as part of DO events. Welander's flip-flop oscillations essentially cover this, except note that the convection also affects the overturning circulation leading to strong MOC variability/ relaxation oscillations.

4.4 Stochastically driven MOC variability

A review of classes of THC oscillations: small amplitude/ large amplitude; linear and stochastically forced/ nonlinear self-sustained; loop oscillations due to advection around the THC path, or periodic switches between convective and non convective states; relaxation oscillations; noise induced switches between steady state, stochastic resonance;

Details of noise-driven THC/MOC variability:

- Linear Loop-oscillations due to advection around the circulation path: Stability regimes in a 4-box model: stable, stable oscillatory, [Hopf bifurcation], unstable oscillatory, unstable; Note changes from 2-box Stommel model: oscillatory behavior and change to the point of instability on the bifurcation diagram; Stochastic forcing can excite this damped oscillatory variability. Next, the GCM study of Delworth et al. (1993); this paper also demonstrates the link between the variability of meridional density gradients and of the THC; Note the proposed role of changes to the gyre circulation in this paper, mention related mechanisms based on ocean mid-latitude Rossby wave propagation; then a box model fit to the GCM, showing that the horizontal gyre variability may not be critical and that the variability is due to the excitation of a damped oscillatory mode (Griffies and Tziperman, 1995); Useful and interesting analysis methods: composites (DMS Figs. 6,7), and regression analysis between scalar indices (Figs. 8,9) and between scalar indices and fields (Figs. 10, 11, 12).

- A complementary view of the above stochastic excitation of damped THC oscillatory mode: first, Hasselmann's model driven by white noise and leading to a red spectrum response (to derive the spectrum of the response, Fourier transform the first equation, and multiply transformed equation and its complex conjugate).

$$\begin{aligned}\dot{x}+\gamma x &= \xi(t)\\ P(\omega)=|\hat{x}|^2 &= \xi_0^2/(\omega^2+\gamma^2).\end{aligned}$$

  Compare this to a damped oscillatory mode excited by noise that results in a spectral peak,

$$\begin{aligned}\ddot{x}+\gamma\dot{x}+\Omega^2 x &= \xi(t)\\ P(\omega)=|\hat{x}|^2 &= \xi_0^2/((\Omega^2-\omega^2)+\gamma^2).\end{aligned}$$

- Stochastic variability due to noise induced transitions between steady states (double potential well Cessi, 1994). A GCM version of jumping between two equilibria under sufficiently

strong stochastic forcing: Weaver and Hughes (1994).
(time permitting:) Cessi: Stommel 2 box model from section 2 with model derivation and in particular getting to eqn 2.9 with temperature fixed and salinity difference satisfying an equation of a particle on a double potential surface; section 3 with deterministic perturbation;

- Stochastic resonance: periodic FW forcing plus noise. Matlab code `Stommel_stochastic_resonance.m` from APM115, and jpeg figures with results: `Stochastic_Resonance_a,b,c.jpg`;

- (Time permitting) THC oscillations due to "Thermal" Rossby waves analyzed by Te Raa and Dijkstra (2002), using equations 8-10 and Figure 7 of Zanna et al. (2011).

- Stochastic forcing, non normal THC dynamics, transient amplification; stochastic optimals if this wasn't discussed in the context of ENSO, see ENSO notes above (e.g., in the context of The 3-box model of Tziperman and Ioannou (2002), or the spatially resolved 2d model of Zanna and Tziperman (2005)). Figures for the amplification and mechanism, taken from a talk on this subject (file nonormal_THC.pdf).

- (Time permitting): A more general issue that comes up in this application of transient amplification is the treatment of singular norm kernel (appendix) and infinite amplification; show and explain the first mechanism of amplification (Figure 2); note how limited the amplification may actually be in this mechanism.

- (Time permitting) Use of eigenvectors for finding the instability mechanism: linearize, solve eigenvalue problem, substitute spatial structure of eigenvalue into equations and see which equations provide the positive/ negative feedbacks; results for THC problem (e.g., Tziperman et al., 1994, section 3): destabilizing role of $v'\nabla\bar{S}$ and stabilizing role of $\bar{v}\nabla S'$; difference in stability mechanism in upper ocean ($v'\nabla\bar{S}$) vs that of the deep ocean (where $\nabla\bar{S} = 0$ and $\bar{v}\nabla S'$ is dominant); for temperature, also ($v'\nabla\bar{T}$ is more important, but from the eigenvectors one can see that v' is dominated by salinity effects; GCM verification and the distance of present-day THC from stability threshold: Figs 4,5,6 from Tziperman et al. (1994); Fig 3 from Toggweiler et al. (1996); Figs 1, 2, 3 from Tziperman (1997).

4.5 More on stochastic variability (time permitting)

- As a preparation for the rest of this class: the derivation of diffusion equation for Brownian motion following Einstein's derivation from Gardiner (1983) section 1.2.1; next, justify the drift term heuristically; then, derivation Fokker-Plank equation from Rodean (1996), chapter 5; Note that equation 5.17 has a typo, where the LHS should be $\frac{\partial}{\partial t}T_\tau(y_3|y_1)$; Then, first passage time for homogeneous processes from Gardiner (1983) section 5.2.7 equations 5.2.139-5.2.150; 5.2.153-5.2.158; then the one absorbing boundary (section b) and explain the relation of this to the escape over the potential barrier, where the potential barrier is actually an absorbing boundary, with equations 5.2.162-5.2.165; Note that 5.2.165 from Gardiner (1983) is identical to equation 4.7 from Cessi (1994); Next, random telegraph processes are explained in Gardiner (1983) section 3.8.5, including the correlation function for such a process; Cessi (1994) takes the Fourier transform of these correlation functions to obtain the spectrum in the limit of large jumps, for which the double well potential problem is similar to the random telegraph problem.

Next, back to Cessi (1994) section 4: equation 4.4 (Fokker-Planck), 4.6 and Fig. 6 (the stationary solution for the pdf); then the expressions for the mean escape time (4.7) and the rest of the equations all the way to end of section 4, including the random telegraph process and the steady probabilities for this process;

Finally, from section 5 of Cessi (1994) with the solutions for the spectrum in the regime of small noise (linearized dynamics) and larger noise (random telegraph); For the solution in the small noise regime (equation 5.3), let $y' = y - y_a$ and then Fourier transform the equation to get $-i\omega\hat{y}' = -V_{yy}\hat{y} + \hat{p}'$ where hat stands for Fourier transform; then write the complex conjugate of this equation, multiply them together using the fact that the spectrum is $\mathcal{S}_a(w) = \hat{y}'\hat{y}'^{\dagger}$ to get equation 5.5; Show the fit to the numerical spectrum of the stochastically driven Stommel model, Figure 7;

- Zonally averaged models and closures to 2d models (Dijkstra, 2000, section 6.6.2, pages 282-286, including technical box 6.3); Atmospheric feedbacks (Marotzke, 1996)?

5 Dansgaard-Oeschger and Heinrich events

Downloads here.

Item numbering in the following outline correspond to file numbering in downloads directory.

1 **Introduction, observed record of Heinrich and DO events:** IRD, Greenland warming events, possible relation between the two; synchronous collapses? or maybe not? Use Figures of obs from Heinrich_slides.pdf

2 **Winton model:** Convection, air-sea and slow diffusion: Relaxation oscillations/ Thermohaline flushes/ "deep decoupling" oscillations (Winton, 1993, section IV).

3 **THC flushes and DO events:** DO explained by large amplitude THC changes (Ganopolski and Rahmstorf, 2001); from this paper, show figs 1,2,3,5: hysteresis diagrams for modern and glacial climates demonstrating the ease of making a transition between the two THC states in glacial climate; time series of THC during DO events; these oscillations are basically the same as Winton's deep decoupling oscillations and flushes (Figure 7 in Winton (1993), see also under THC variability).

4 **Alternatively, sea ice as an amplifier of small amplitude THC variability:** Preliminaries: sea ice albedo and insulating feedbacks; volume vs area in present-day climate (i.e., typical sea ice thickness in Arctic and Southern Ocean); simple model equation for sea ice volume (eqn numbers from Sayag et al., 2004): sea ice melting and formation (18), short

wave induced melting (19 and 3rd term on rhs in 20), sea ice volume equation (20); climate feedbacks: insulating feedback (3), albedo feedback (19).

Fig. 1 from Li et al. (2005) AGCM experiments (again Heinrich_slides.pdf). Again sea ice as an amplifier of *small* THC variability (Kaspi et al., 2004); Possible variants of the sea ice amplification idea: Stochastic excitation of THC+sea ice leads to DO-like variability (Fig. 5 in Timmermann et al., 2003); similarly, self-sustained DO events with sea ice amplification in Loving and Vallis (2005), including figures on pages 16, 22, 24 in pdf.

5 **Precise clock behind DO events? Stochastic resonance?** First, figs 1,2 from Rahmstorf (2003): (time permitting: clock error, triggering error and dating error); is it significant, or does the fact that we are free to look for a periodicity for which some "clock" might fit the time series makes it more likely for the time series to seem as if it is driven by a precise clock? Next, stochastic resonance: (Alley et al., 2001): consider a histogram of waiting time between DO events (Fig. 2) and find that these are multiples of 1470, suggesting stochastic resonance as a possible explanation. The bad news: no clock, (Ditlevsen et al., 2007), see their Fig. 1 and read their very short conclusions section.

6 **DO teleconnections:** Wang et al. (2001) show a strong correlation of Hulu cave in China with Greenland ice cores (Fig. 1). Similarly, Denton and Hendy (1994) show a correlation of Younger Dryas and glaciers in New Zealand. What is the mechanism? (1) Atmospheric teleconnections: if this wasn't covered in the El Nino section, discuss atmospheric Rossby wave teleconnections. (2) MOC/THC teleconnections: weaker NADW import to Southern Ocean due weakening of MOC (Manabe and Stouffer, 1995); see also the related THC seesaw (Broecker (1998), and this). (3) Alternatively, teleconnections may be via ocean wave motions e.g., Johnson and Marshall (2004) Fig. 4 and discussion around eqns 1,2,10): essentially equatorial and coastal Kelvin waves propagate anomalies due to changes in convection rate very fast from the north-west Atlantic and spread the information along eastern boundaries, and then slower Rossby waves transmit the information to the ocean interior on a decadal time scale. This mechanism also allows for inter-basin exchanges.

7 **Heinrich events: MacAyeal's binge-purge mechanism:** start with MacAyeal (1993a)'s, argument that external forcing is not likely to play a role (section 2, eqns 1-5, p 777); heuristic argument for the time scale (section 5, p 782, eqns 19-25, note that lhs of eqn 23 should be $\tilde{\theta}_y(0,t)$). Show equations for the more detailed model of MacAyeal (1993b) (from slide 22 of Heinrich_slides.pdf or Kaspi et al., 2004), and numerical solution for temperature and ice height (slide 11), explaining all temperature maxima and minima of temperature during the cycle.

8 On the relation between DO and Heinrich events: do major DO follows Heinrich events? Bond cycle (image on course web page is from here)? Do Heinrich events happen during a cold period just before DO events? When the ice sheet model is coupled to a simple ocean-atmosphere model (slide 21), can get the response of the climate system as well (slides 27,28,29) via a THC shutdown (cold event) followed by a flush (warm event).

9 Finally, synchronous collapses? slides (33-47).

6 Glacial cycles

Downloads here.

6.1 Basics

1 Briefly: glacial cycle phenomenology: SIS intro (slides 1-7)

2 Basic ingredients that will later be incorporated into different glacial theories (lecture 8 from WH notes),

1. energy balance and albedo feeback
2. ice sheet dynamics and Glenn's law (go over details below)
3. parabolic profile (go over details below)
4. accumulation and ablation as function of ice sheet height
5. equilibrium line
6. ice streams
7. calving
8. dust loading and enhanced ablation
9. temperature-precipitation feedback
10. isostatic adjustment
11. Milankovitch forcing
12. geothermal heating

3 Intro to ice dynamics: rate of strain definition from Kundu and Cohen (2002) sections 3.6 and 3.7, pages 56-58; stress and deviatoric stress and stress-rate of strain relationship and Glenn's law from section 2.3, pages 13-15 of Van-Der-Veen (1999).

4 Parabolic profile of ice sheets: first simple version assuming ice is a plastic material, from WH notes p 101. Then with the more accurate expression shown by the solid line in Fig 11.4 in Paterson (1994): the following derivation is especially sloppy in dealing with the constants of integration, and very roughly follows Chapter 5, p 243 eqns 6-10 and p 251, eqns 18-22 from Paterson (1994),

$$\dot{\varepsilon}_{xz} = \frac{1}{2}\frac{du}{dz} = A\tau_{xz}^{n} = A(\rho g(h-z)\frac{dh}{dx})^{n}$$

integrate from $z = 0$ to z, and use the b.c. $u(z = 0) = u_b$,

$$u(z) - u_b = 2A(\rho g \frac{dh}{dx})^n \frac{(h-z)^{n+1}}{n+1} - 2A(\rho g \frac{dh}{dx})^n \frac{h^{n+1}}{n+1}$$

Let $u_b = 0$ (no sliding) and average the velocity in z,

$$\begin{aligned}
\bar{u} &= (1/h) \int_0^h dz 2A \left(\rho g \frac{dh}{dx}\right)^n \frac{(h-z)^{n+1}}{n+1} - 2A \left(\rho g \frac{dh}{dx}\right)^n \left(\frac{h^{n+1}}{n+1}\right) \\
&= \frac{2A}{(n+1)} \left(\rho g h \frac{dh}{dx}\right)^n h \left(\frac{1}{n+2} - 1\right) \\
&= -\frac{2A}{(n+2)} \left(\rho g h \frac{dh}{dx}\right)^n h. \qquad (1)
\end{aligned}$$

Next we use continuity, assuming a constant accumulation of ice at the surface, $d(h\bar{u})/dx = c$ which implies together with the last equation

$$cx + K_1 = h\bar{u} = -\frac{2A}{(n+2)} \left(\rho g h \frac{dh}{dx}\right)^n h^2 = K_2 \left(h \frac{dh}{dx}\right)^n h^2$$

where ablation is assumed to occur only at the edge of the ice sheet at $x = L$. The last eqn may be written as

$$(K_3 x + K_4)^{1/n} dx = h^{2/n+1} dh$$

and solved using boundary conditions of $h(x = 0) = H$ and $h(x = L) = 0$ to obtain

$$(x/L)^{1+1/n} + (h/H)^{2/n+2} = 1.$$

This last equation provides the better fit to obs in Paterson Fig 11.4 (also shown in WH notes).

6.2 Milankovitch

- sea_ice_switch_and_pacing_glacial_cycles.pptx slide 6; Paillard (2001) Fig 2.
- Muller and MacDonald (2002) Chapter 2: sections 2.1 (only until and not including 2.1.1); 2.2 (only 2.2.1-2.2.5).
- Some additions: precession effect is anti-symmetric with respect to seasons and hemispheres, and annual average of precession vanishes at each latitude; precession has no effect when eccentricity is zero (circular orbit); Paillard (2001) p 328: "energy received at a given latitude and between two given orbital positions (for example, between the summer solstice and the autumnal equinox) does not depend on the climatic precession. However, the time necessary for the Earth to move between these two orbital positions (for example, the length

of the summer season) does change with climatic precession. The insolation, defined as the amount of energy received per unit time, therefore changes with climatic precession, but only through the lengths of the seasons."

- Obliquity does effect annual mean at a given latitude, but not global average. Larger obliquity leads to more radiation at the poles in summers, but still none at winter, so more generally high latitude annual insolation depends on obliquity.
- Paillard (2001) Figs. 3,4,5
- Animations and images in web page by Peter Huybers (use Safari), people.fas.harvard.edu/ phuybers/Inso
- Integrated insolation: The 41kyr problem, positive degree days as a motivation for integrating insolation beyond a threshold corresponding to melting point; cancellation of precession because summer intensity and duration are exactly out of phase (Kepler Laws). Show Huybers-integrated-insolation.ppt slides 5,7,9,10.

6.3 Glacial cycle mechanisms

General outline:

1. Temperature-precipitation feedback (WH section 9.1.1)
2. isostatic adjustment (WH section 9.1.2)
3. Adhemar's model, Croll's model (Fig. 1, Paillard, 2001),
4. Milankovitch: a correct calculation of orbital parameters and insolation, and realization that it's the summers that matter.
5. Calder's model (eqn on p. 332 and Fig. 9 of Paillard, 2001),
6. Imbrie and Imbrie (eqn on p. 333 and Fig. 10 of Paillard, 2001),
7. Paillard's model (sec 3.3, eqn on p. 339 and Figs 12 and 13 of Paillard, 2001),
8. Le-Treut and Ghil Le-Treut and Ghil (1983), Rial (1999), (both from WH notes).
9. Stochastic resonance
10. 100 kyr from a collapse of the ice sheets due to geothermal heat build up and induced basal melting (Huybers and Tziperman, 2008)
11. Saltzman "earth system" models (Pacing slides): raise the question of whether we are trying to merely fit the record or explain the mechanism? (WH lecture, including excellent fit to wrong CO_2 record)

12. Pollard (1982): figure 72 from WH notes, adding components and feedbacks until a good fit to ice volume is obtained.

13. Sea ice switch (SIS slides)

14. Phase locking, and a discussion of how a good fit to ice volume does not imply a correct physical mechanism; difference between locking to periodic and quasi-periodic forcing (Pacing slides)

15. Huybers' integrated insolation and the 41kyr problem (Huybers slides).

Some additional details follow.

On the expected spectral characteristics of the solutions to equations 36 and 37 (Imbrie&Imbrie) from lecture 9 of the WH notes: Fourier transforming the two, we find $i\omega\hat{V}(\omega) = -k\hat{i}(\omega)$. Multiply by the complex conjugate of this equation to get the power spectrum $|\hat{V}(\omega)|^2 = k^2|\hat{i}(\omega)|^2/\omega^2$, implying that higher frequency are damped, which emphasizes the low frequencies, including 100kyr, relative to the high ones, including 20 and 41kyr. For the second model, in the simpler case where τ is constant, we similarly have $(i\omega+\tau^{-1})\hat{V}(\omega) = \hat{i}(\omega)$, leading to the spectrum $|\hat{V}(\omega)|^2 = |\hat{i}(\omega)|^2/(\omega^2+\tau^{-2})$, and now τ may be used as a tuning parameter to determine which frequencies are damped. But this still amplifies both the 100 and 400 frequencies in $\hat{i}(\omega)$, which is inconsistent with the proxies.

6.4 CO_2

1. Basics of ocean carbonate system, including carbonate ions, prognostic equations for alkalinity, total carbon and a nutrient, from notes. Section 4 till eqn 9; section 4.1, eqns 10-19; section 4.2 and 4.3 with an approximate analytic solution of the carbonate system. Then use the solution to understand the response of the carbonate system to adding CO_2 from volcanoes, of photosynthesis and remineralization ($2CO_2+2H_2O \rightleftharpoons 2CH_2O+2O_2$) and of $CaCO_3$ deposition or dissolution ($CaCO_3 \rightleftharpoons Ca^{2+}+CO_3^{2-}$, or, equivalently, $CaCO_3+CO_2+H_2O \rightleftharpoons Ca^{2+}+2HCO_3^-$). Note that instead of specifying total CO_2 and alkalinity, we could solve for the carbonate system given any two of (total carbon, alkalinity, pH), this is also useful when trying to reconstruct past CO_2 from proxies that reflect past pH. Then note that we need advection-diffusion equations for the total carbon and alkalinity (section 5, first paragraph).

2. (time permitting) The exact solution of this set of equation for pCO_2 as function of total CO_2 and alkalinity based on Fig 1.1.3 from Zeebe's book.

3. The 3 box model of Toggweiler (1999) showing how CO_2 can change in response to change in mixing in the Southern Ocean between surface water and NADW below (notes based on Toggweiler's eqns 4, 5, 8).

4. Criticism of the results: same notes, (based on Toggweiler's section 2, paragraphs 2, 3 on left column, page 575). Bottom line is that the model also predicts changes to the high latitude surface nutrients PO_{4h}, and this change hasn't been observed. Toggweiler later shows in his

paper that reversing the THC in the Southern Ocean (to be more realistic, actually) helps with this.

5. (time permitting) Results of full 3 box model for glacial CO_2 as function of ventilation by f_{dh} and high latitude biological pump P_h (section 2), Figures 2 and 3.

7 Pliocene climate

Phenomenology (Molnar and Cane, 2007; Dowsett et al., 2010) and relevant proxies for temperature, productivity, upwelling, CO_2, etc. Pliocene as nearest analogue of future warming (although only for the equilibrium response). Observations of global temperature, permanent El Nino and warming of mid-latitude upwelling sites.

Possible mechanisms for permanent El Nino: FW flux causing a collapse of the meridional density gradient and therefore meridional ocean heat flux, which leads to a deepening of the thermocline; Hurricanes and tropical mixing; opening of central American seaway; movement of new Guinea and changes in Pacific-Indian water mass exchange; atmospheric superrotation, possibly driven by strengthening MJO. Mechanisms for warming of upwelling sites.

Details of all of the above in Pliocene powerpoint presentation.

8 Equable climate

Downloads here.

Earth Climate was exceptionally warm, and the equator to pole temperature difference (EPTD) exceptionally small, during the Eocene (55Myr ago), when continental configuration was not dramatically different from present-day. Many explanations have been proposed, and we will briefly survey some.

First, Phenomenology and relevant proxies from slides.

8.1 Equator to pole Hadley cell

(Farrell, 1990). The idea briefly: Angular momentum conservation leading to large u in the upper branch of the Hadley cell: $M = (u + \Omega r \cos\theta) r \cos\theta$ and if a particle starts with $u(\text{equator}) = 0$, we find from $M(30) = M(equator)$ that $u(30) = (6,300,000 \times 2\pi/(24 \times 3600)) \times (1/\cos(30) - \cos(30)) = 132 m/sec$. The resulting large u_z is balanced via thermal wind by strong T_y, leading to a large EPTD (eqn 1.5 in Farrell (1990)). To break this constraint, can dissipate some angular momentum, reduce f (as on Venus), or increase the tropopause height H which appears in the solution for the edge of the Hadley cell.

- Start with the friction-less theory from Vallis (2006), highlighted parts of sections 11.2.2-11.2.3.

- Then cover highlighted parts of Farrell (1990), which is based on an extension of Held and Hou (1980) to include dissipation, based on Hou (1984).

8.2 Polar stratospheric clouds (PSCs)

PSCs are formed in the lower stratosphere (15-25 km) at temperatures below -78°C. They are optically thick and high, and therefore have a significant greenhouse effect. Water ice, which is a major ingredient of PSCs, is formed in the stratosphere via methane oxidation. Methane, unlike water vapor, is able to get past the tropopause "cold trap" given its freezing point of -182°C. Sloan et al. (1992) proposed that PSCs may have contributed to equable climate conditions, and Kirk-davidoff et al. (2002) suggested a positive feedback that would enhance the formation of PSCs in a warm climate. Details follow.

- **Basics:** QG potential vorticity, EP fluxes, transformed Eulerian mean equations (Vallis).

- **Planetary wave forcing:** Topographically forced vertically propagating planetary waves, conditions on mean zonal flow that lead to vertical propagation (Vallis).

- **Zonal stratospheric circulation:** (Vallis) SW absorption near summer pole leads to a reversed temperature gradient in summer hemisphere: i.e., $T_y < 0$ in southern hemisphere during Jan, and $T_y > 0$ in northern hemisphere during July. Winter hemisphere (northern Jan, southern July) has no SW at pole, temperature gradient is not reversed. During southern winter (July), therefore, $T_y > 0$, and therefore $u_z \propto \rho_y / f_0 \propto -T_y / f_0 > 0$. Using $u = 0$ at top of stratosphere we get $u < 0$ (easterlies) during summer (Jan) in the southern hemisphere stratosphere. Similarly, $u < 0$ (easterlies) during summer (July) in northern hemisphere. During winter stratospheric zonal winds are westerlies. Note that stationary Rossby waves cannot propagate from the troposphere into easterlies, therefore can only reach the stratosphere in the winter hemisphere.

- **Brewer-Dobson stratospheric circulation:** (Vallis) zonally averaged Transformed Eulerian Mean (TEM) momentum balance derived above is $-f_0 \overline{v}^* = \overline{v'q'}$. Assuming the potential vorticity flux is down gradient (equatorward, because the gradient is dominated by β), the rhs is negative, so that the mean flow $\overline{v}^* > 0$ is poleward. B-D circulation warms the pole and cools the equator in the stratosphere (Vallis eqn 13.89): $N^2 \overline{w}^* = \frac{\theta_E - \theta}{\tau}$, together with positive w in tropics and negative in polar areas forced by poleward B-D meridional flow. This leads to $\theta < \theta_E$ (cooling!) at equator (where $\overline{w}^* > 0$) and $\theta > \theta_E$ (warming!) at pole (where $\overline{w}^* < 0$).

- **Feedback between EPTD, vertically propagating planetary waves, Brewer-Dobson stratospheric circulation and PSCs:** (Kirk-davidoff et al., 2002) warmer climate means weaker tropospheric EPTD, this leads to weaker mean tropospheric winds and weaker synoptic scale motions (which are, in turn, created via baroclinic instability of the mean winds and meridional temperature gradient). Both of these factors weaken the production of vertically propagating Rossby waves (forced by mean winds interacting with topography, and by synoptic motions). As a result, weaker Eliassen-Palm flux EP, weaker $\nabla \cdot EP = \overline{v'q'}$, weaker B-D circulation, and therefore colder pole and warmer equator. Colder pole allows more PSCs to develop. This, in turn, further weakens the EPTD in troposphere, providing a positive feedback. However, see Korty and Emanuel (2007).

8.3 Hurricanes and ocean mixing

- What sets maximum hurricane strength, "hyper-canes" (from Kerry's web page): rate of energy *input* per unit area into the hurricane is roughly $G = \varepsilon C_k \rho V_s L(q_0^* - q_a)$, where V_s is the max surface wind speed, ε efficiency in translating enthalpy to K.E., qs are the atmospheric surface specific humidity ocean saturation humidity, L is the latent heat of evaporation, ρ the air density and C_k the bulk coefficient for evaporation. The rate of energy dissipation per unit area is given by $D = C_D \rho V_s^3$.

- Think of the hurricane as a Carnot cycle: air acquires heat (in the form of moisture from evaporation) as it flows along the surface toward the center, it then expands adiabatically while releasing latent heat going up; it releases the heat to the environment while mixing out of the convective plumes at the top of the storm (temperature T_0, and undergoes compression at while descending back to the surface. The *efficiency* of KE generation in a Carnot cycle in terms of the temperatures of the warm and cold reservoirs involved is $\varepsilon = (T_H - T_C)/T_H$. For Hurricanes, $T_H = SST$ is temperature of the heat source (the ocean surface). $T_C = T_0$ is the average temperature at which heat is lost by the air parcels at the top of the storm. The taller a hurricane is, the lower the temperature T_0 at its top and thus, the greater the thermodynamic efficiency. For a typical hurricane, $\varepsilon \approx 1/3$.

- Setting dissipation equal to generation ($G = D$), we get $V_s^2 = \varepsilon L(q_0^* - q_a)C_k/C_D$. Assuming the atmosphere to be 85% saturated, and the ratio of the two bulk coefficients to be about one, we get
$$V_s^2 = \varepsilon L 0.15 q_0^* = \frac{SST - T_0}{SST} L 0.15 q_0^*.$$
Note that this is exponential in temperature, because of the Clausius-Clapeyron relationship.

- Finally, assuming that stronger hurricanes lead to stronger ocean mixing, and this to stronger MOC. Stronger MOC means warmer poles (Emanuel, 2002).

- Consequences on EPTD of enhanced tropical ocean mixing

8.4 Convective cloud feedback

- Energy balance calculation for Arctic comparing options for warming the Arctic.

- Results of box model, hysteresis and multiple equilibria, from Powerpoint.

- Moist adiabatic lapse rate from my notes based on Marshall and Plumb (2008).

- Using MSE to diagnose convective stability, again from my notes.
- Two level model, section 2 of Abbot and Tziperman (2009) demonstrating the multiple equilibria and hysteresis analytically.
- Supporting results from more complex models: SCAM, IPCC, SP-CESM, from Powerpoint presentation.

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