On the abyssal circulation of the world ocean – II. An idealized model of the circulation pattern and amplitude in oceanic basins

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Abstract—Stationary planetary flow patterns driven by source sink distributions in an ocean on the rotating earth were developed in Part I. These theoretical results are now used to construct a highly idealized model of the general abyssal circulation of the world ocean. The model is based on the postulation of the existence of two concentrated sources of abyssal waters (one in the North Atlantic and another in the Weddell Sea) and on a uniformly distributed upward flux of water from the abyssal to the upper layers as part of the mechanism of the main oceanic thermocline. Order of magnitude calculations based on this model lead to a variety of estimates of the time in which the deep water is replaced (from every 200 to 1800 years). Comparison of leading terms in the dynamical equations and equation describing the flux divergence of a tracer shows that there is a large range of lateral eddy coefficient which will influence the distribution of the tracer but not affect the dynamically determined planetary flow patterns.

INTRODUCTION

IN Part I of this series of papers (STOMMEL and ARONS, 1959) we outlined the manner in which stationary flow patterns in oceanic basins on a spherical earth might be predicted for various source-sink distributions. These predictions were based on a previously published theory and interpretation of rotating basin experiments (STOMMEL *et al.*, 1958). In this paper we use the preceding results and a theory of the oceanic thermocline (ROBINSON and STOMMEL, 1959) to develop a model of the general abyssal circulation of the world ocean. The model, is of course, highly idealized, but we feel that it affords a plausible starting point for visualizing certain broad, qualitative features of the circulation, and that it may help generate suggestions for experiments and observations that will provide tests of the validity of the concepts which have been used.

OUTLINE OF THE MODEL

WÜST (1938) first presented a chart of the potential temperature at the bottom for the whole ocean, and showed that this clearly indicated that there was a powerful source of low temperature water in the Weddell Sea, and that this water spreads northwards along the western side of each ocean basin. This chart has become a standard exhibit in oceanographic textbooks, and, taken by itself, would suggest that on the basis of temperature alone all the deepest water in the ocean originates mainly in the Weddell Sea.

Of course, WÜST did not mean to suggest that there was only one source, and in a later paper (1951) he explicitly showed the great influence of the waters of the North Atlantic – through their high salinity – on the rest of the deep ocean water. This is also shown by the study of the T-S relations of deep water (STOMMEL, 1958) and by a study of the charts presented in Figs. 1(a) and 1(b), which we have compiled partly from WÜST's similar chart of the Atlantic, and partly from a new analysis of data in the Indian and Pacific Oceans. (The spacing of isohalines in Fig. 1(a) is not uniform, but is laid out in geometric progression to prevent extreme crowding of lines in the North Atlantic). One can see that there is a very high salinity tongue extending from the Straits of Gibraltar, down the Atlantic and then eastward around Antarctica, and finally extending up northward into the Indian and Pacific Oceans – the salinity falling all the way. This chart serves to emphasize the importance of the contribution of North Atlantic Deep Water to the general abyssal circulation. The Mediterranean outflow is not massive enough to warrant inclusion as a third mass source. Its high salinity, however, may be regarded as supplying a 'tracer' by means of which the flow of water from the greater North Atlantic mass source can be followed.

Another figure which helps to visualize the sources of the two abyssal water components and the way in which they sink down and flow around Antarctica and thence up into the Indian and Pacific Oceans is presented in Figs. 2(a) and 2(b)*. Thus in Fig. 2(a) several vertical profiles of salinity are sketched to show the spread and vertical extent of the water at the salinity maximum already depicted in Figs. 1(a) and 1(b). To emphasize the features under description only two isohalines are shown: 34·70‰ and 34·76‰, although of course the total range of salinity variation in the ocean is very much greater than this narrow range. In the North Atlantic profile in Fig. 2(a), the entire profile is filled with water of salinity greater than 34·76‰. The North Atlantic is the source of the high salinity Deep Water. In the South Atlantic the high salinity at the surface is separated from that in the Deep Water by a layer of fresh Antarctic Intermediate Water.

We can trace the high salinity Deep Water all around Antarctica – but we note it does not extend all the way to the bottom at the very foot of the Antarctic Continent. A somewhat diluted portion (between 34·70 and 34·76‰) extends up into the Indian and Pacific Oceans at depth. The high salinities shown at the surface in the Pacific Ocean do not connect anywhere with the deep maximum. In the Indian Ocean the picture is somewhat more complex. There is a flow of saline water from the Red Sea which influences the salinity in the Deep Water of Equatorial and North Indian Ocean, and masks the influence of the North Atlantic component on the salinity there.

In Fig. 2(b), two isotherms are plotted on the same profile: 0°C and 2°C. The very cold water, less than 0 °C, originates at the surface in the Weddell Sea, and extends eastward around Antarctica, vanishing about at the Ross Sea, and never extending very far north. A more dilute portion, between 0°C and 2°C reaches further northward: in the Atlantic it occupies only a small portion of the deepest part of the western trough of the South Atlantic, but does not extend into the North Atlantic. This cold water occupies all the deep regions of the Pacific and Indian Oceans. In the North Indian Ocean it is clear that the Red Sea Water does not extend into the Deep Water on account of the temperature – although the salinity profile in Fig. 2(a) is ambiguous on this point.

In making the above description of the distribution of properties we have purposely used words like 'extends' instead of 'flows,' and 'occupies' instead of 'fills up,' in order to avoid implying that a process of actual flow is occurring. Actually, of course, the inference we wish to draw from the distributions of salinity and temperature in

*Previously unpublished, but constructed by E. D. STROUP and H. STOMMEL in the summer of 1958.

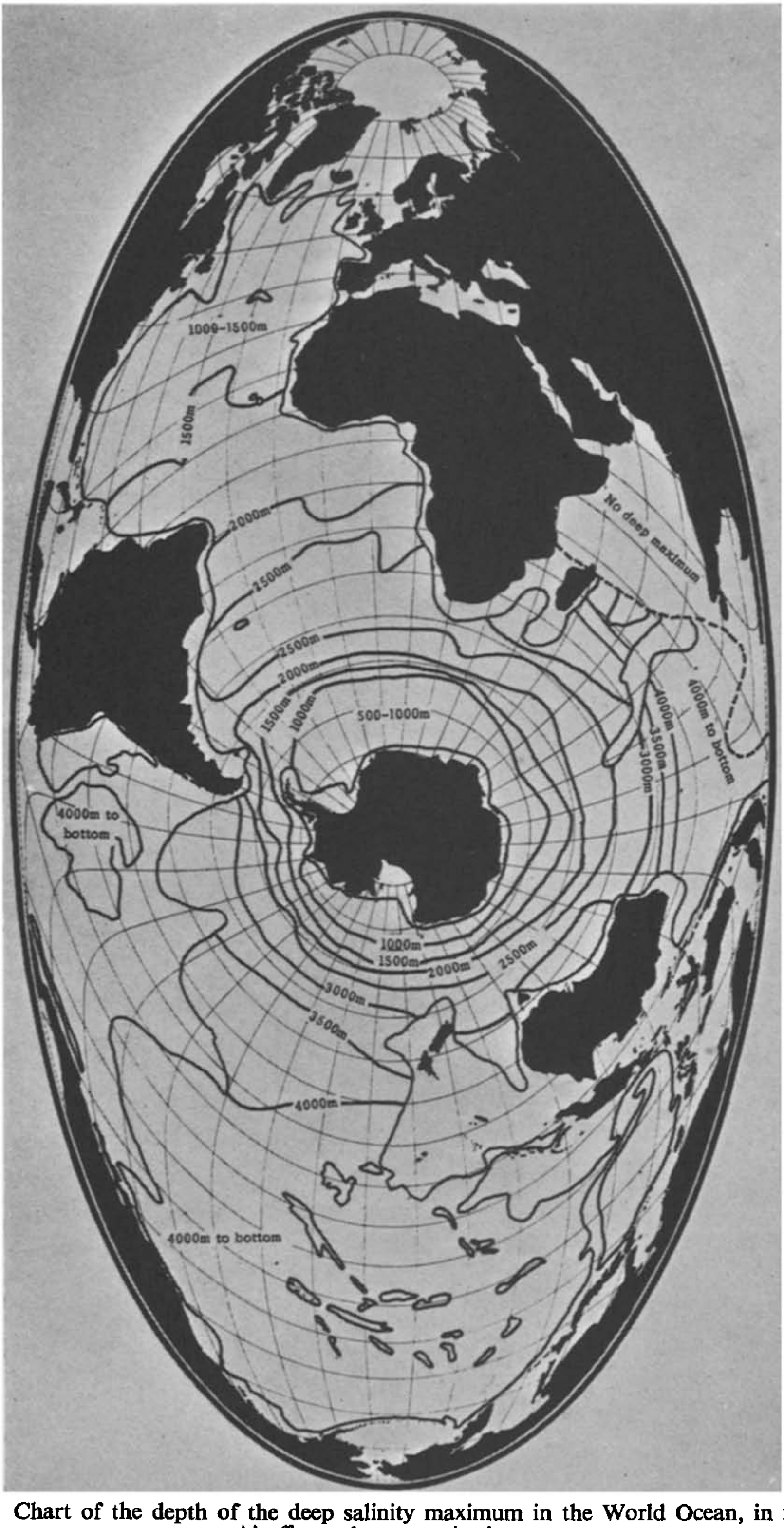

FIG. 1(a). Chart of the depth of the deep salinity maximum in the World Ocean, in metres. Aitoff equal-area projection.

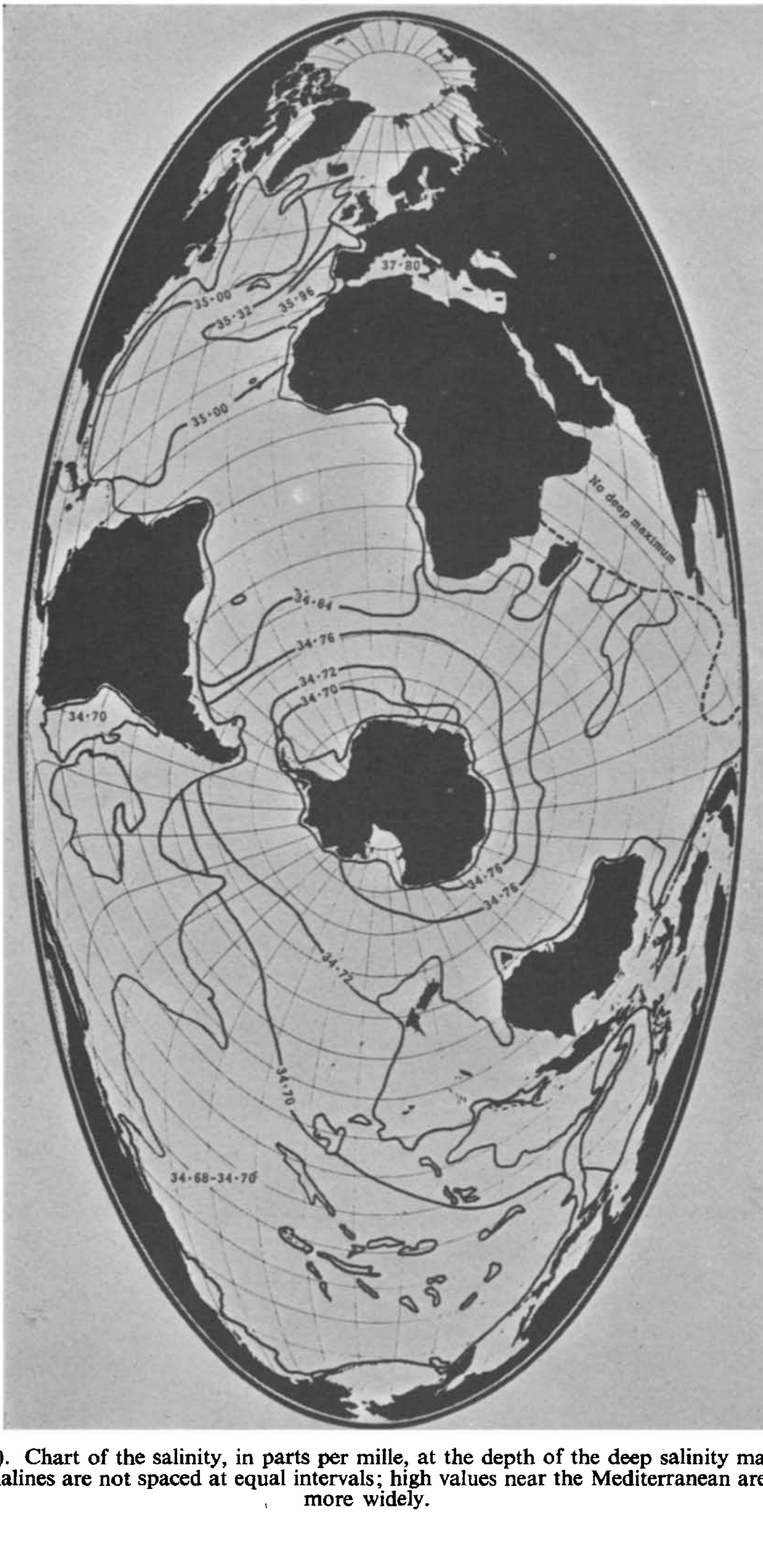

Fig. 1(b). Chart of the salinity, in parts per mille, at the depth of the deep salinity maximum. The isohalines are not spaced at equal intervals; high values near the Mediterranean are spaced more widely.

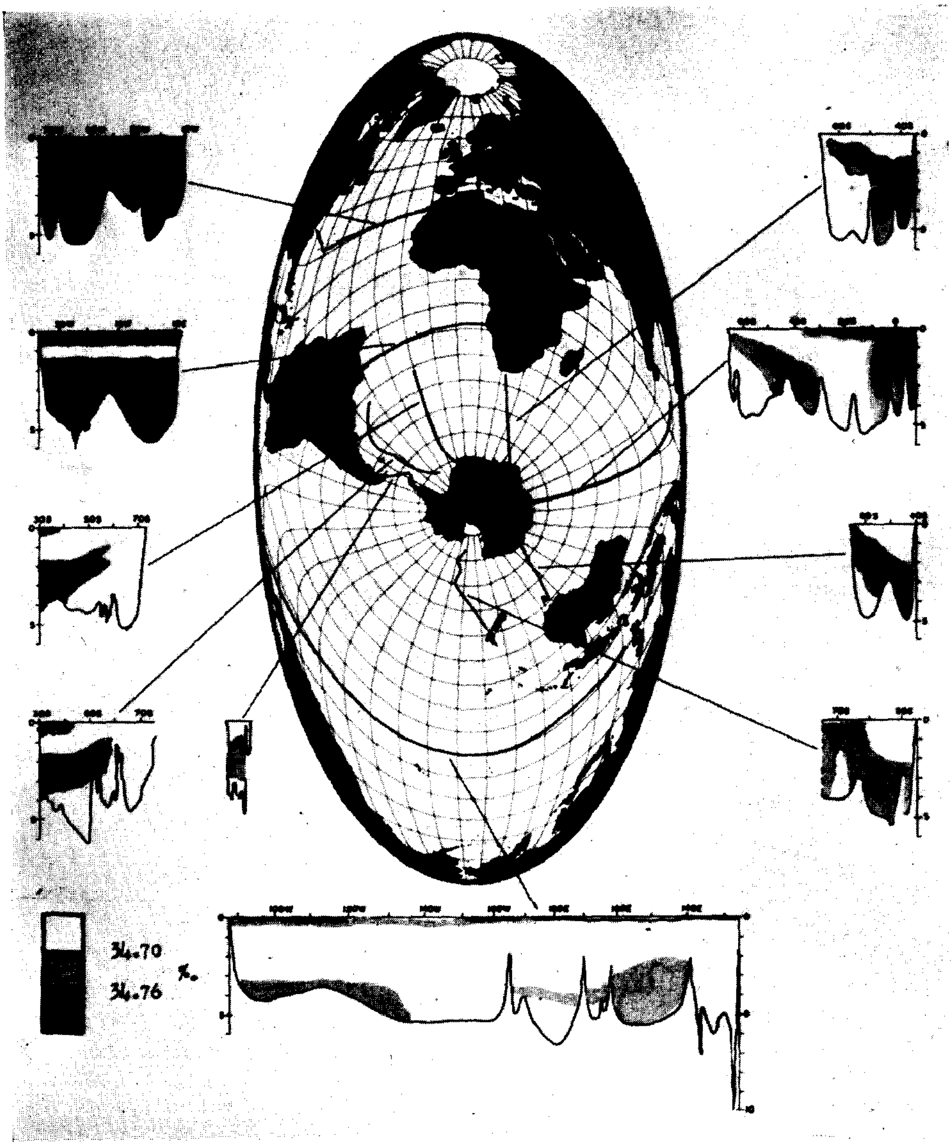

FIG. 2(a). Sketch of distribution of salinity on vertical profiles using only two selected isohalines chosen to depict in particular the influence of North Atlantic Deep Water on the Deep and Bottom Water characteristics of the rest of the World Ocean.

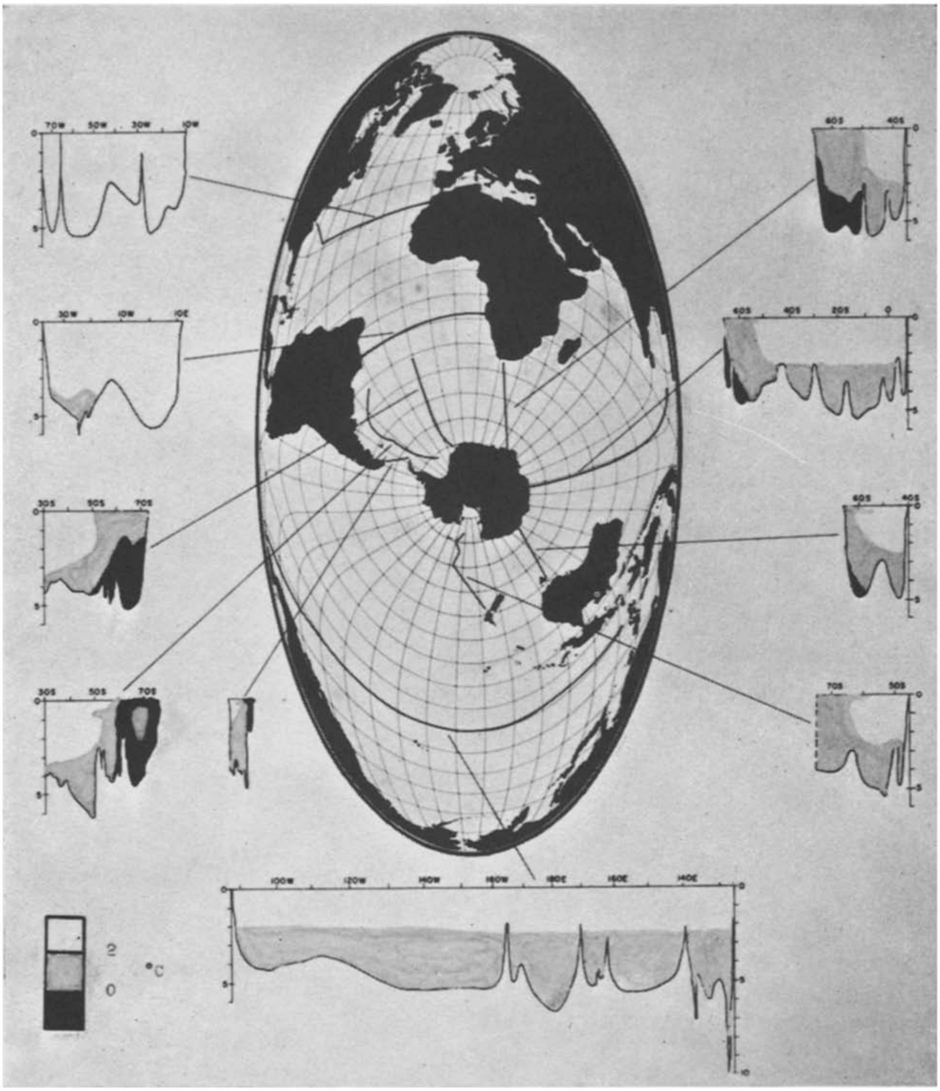

FIG. 2(b). Sketch of distribution of temperature on vertical profiles using only two isotherms chosen to depict the influence of cold Bottom Water from the Weddell Sea on the temperature of the Deep and Bottom Waters of the World Ocean.

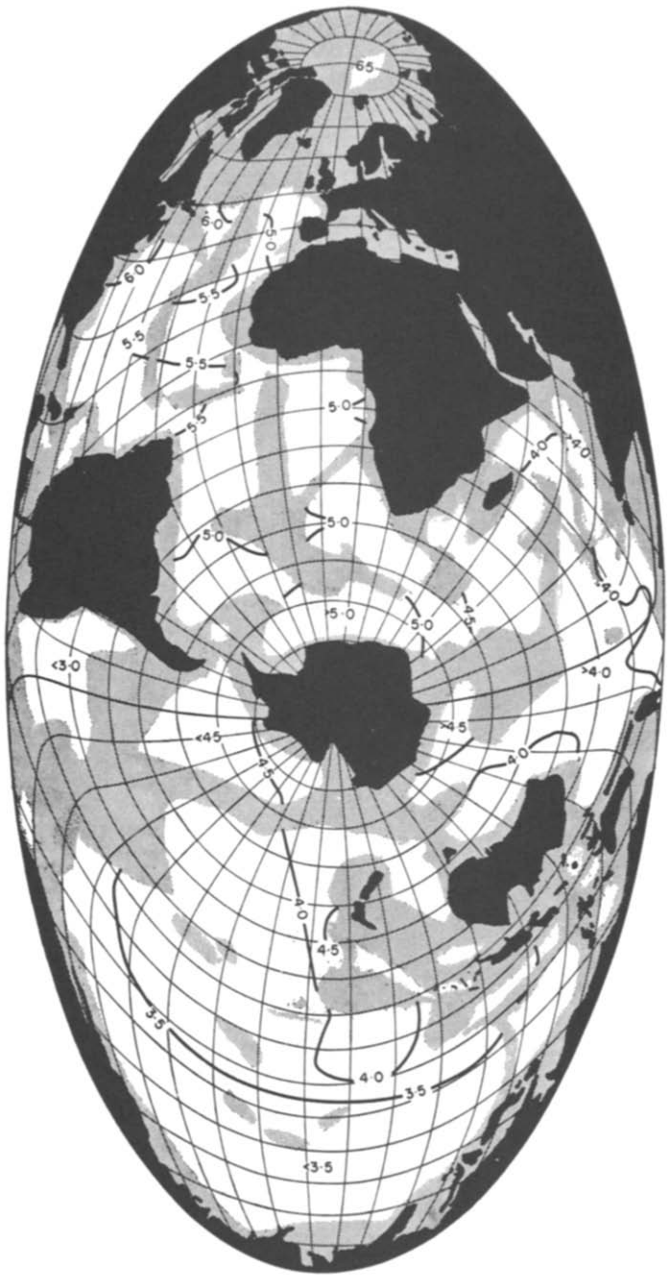

Fig. 3. Sketch of the distribution of dissolved oxygen (ml/l) in the World Ocean at a depth of 4000 m., showing elevated oxygen values in the western North Atlantic and near the Weddell Sea. The oxygen is depleted in regions far removed from these two source areas.

the deep waters is that they are evidence of flow, and this is the inference oceanographers have traditionally made on the basis of the tongue-like distribution of properties. But, as we will see, the tongue-like distribution of properties indicates flow only in a very general way and not in detail. Thus we believe that the distribution of properties merely helps us to indicate the location of the sources of the deep water and general direction of flow from one ocean basin to another. However, we believe that the detailed structure of the mean currents in individual ocean basins – the strong western boundary currents, and large cyclonic gyres – is not shown clearly by the distribution of temperature and salinity because of a masking effect due to lateral mixing – a subject into which we will enquire further in Section 5 of this paper, and will take up *in extenso* in Part III.

A chart of the concentration of dissolved oxygen at 4000 m is presented in Fig. 3. This chart is a preliminary edition of a set of deep ocean charts now in preparation by E. D. STROUP and STOMMEL, and is based upon the totality of deep oxygen measurements available. It shows the high oxygen content of the North Atlantic Deep Water, and the secondary maximum of oxygen in the neighbourhood of the Weddell Sea. Throughout all the rest of the ocean, the deep water oxygen decreases. Quite evidently there is no source of deep water in the Indian or Pacific Oceans.

Thus, in the light of the oxygen data, we assume that bottom water is generated by cooling in only two locations: (a) in the North Atlantic, (b) in the Weddell Sea. If in addition we assume that the mechanism of the oceanic thermocline involves a general upward motion of water at mid-depths, the theory presented in Part I leads to a system of boundary currents and interior geostrophic flow of the qualitative pattern sketched in Fig. 4.

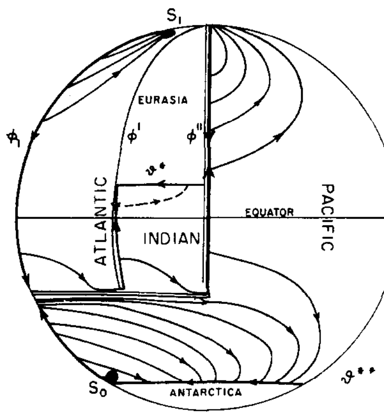

FIG. 4. A circulation diagram showing theoretical distribution of abyssal currents proceeding from the two sources S_0 and S_1 and flowing into a diffuse sink uniformly distributed over the whole ocean area.

Fig. 4 shows a sketch of a global ocean with meridional boundaries at ϕ_1, ϕ_2, ϕ' and ϕ'' and latitudinal boundaries at ϑ^*, ϑ^{**}. Thus we have very roughly a picture which looks like the real ocean basins. On the left is the Atlantic. The barrier ϕ' extending across the equator to mid-latitudes is the European-African land mass. The barrier ϕ'' is the East-Asian coast, extending down across the equator to Australia and New Zealand (in fact, of course this is not completely impermeable to water flow. The land mass bounded by ϕ', ϕ'' and ϑ^* is the Asian Continent. South of it we find the Indian Ocean. From ϕ'' to ϕ_2 we have the Pacific Ocean. The barriers ϕ_1 and ϕ_2 are the eastern and western edges of the North and South American Continental land mass drawn straight across Drake Passage to Antarctica. South of latitude ϑ^{**} lies Antarctica. Source S_0 is the source of Antarctic Bottom Water; Source S_1 is the source of North Atlantic Deep Water.

Isobars and schematic western boundary currents are sketched according to the principles described in Part I of this series – assuming a general upward motion over the entire ocean at mid-depths. From the North Atlantic source S_1 the water flows southward in a western boundary current. Part of this current feeds an interior geostrophic flow in the North Atlantic to the east and north-east. Some of this water rises vertically through the thermocline and the rest eventually rejoins the boundary current in higher latitudes nearer to the cource, thus forming a North Atlantic abyssal gyre.

It is well known that the existence of the southward flowing boundary current beneath the Gulf Stream was verified in the March–April 1957 *Discovery II – Atlantis* Expedition. Most of the North Atlantic Deep Water crosses the equator in the main strength of the western boundary current down the coast of South America past Brazil and Argentina to about 35 °S latitude. Along this course more water is lost to an interior geostrophic flow to the east and south-east, but we hypothesize that this water does not form a closed South Atlantic abyssal gyre, because upon reaching the latitude 35 °S it joins a zonal current flowing eastward toward the Cape of Good Hope. (See Section 7 and Fig. 16 of Part I). The zone bounded by 35°S and 55°S is marked by currents that flow all around the world; it is the zone in which communication between the Atlantic, Indian and Pacific Oceans is constrained to occur by the geographical distribution of the continents. We hypothesize that the Antarctic Bottom Water from the source S_0 flows northward along the western edge of the Weddell Sea to the 35 ° – 55 °S zone, where it veers eastward, joining the backing N Atlantic component, to form a great zonal stream that carries Atlantic water into both the Indian and Pacific Oceans. Upon passing the Cape of Good Hope some of this current flows northward in a narrow western boundary current that feeds the abyssal circulation of the entire Indian Ocean. The remainder of the zonal stream flows eastward, and after passing south of New Zealand, flows northward into the Pacific Ocean as a narrow western boundary current along the Tonga–Kermadec Trench. After crossing the equator the western boundary current flows northward to a latitude of about 30° $\pm$ 10°N where it exhausts itself. Analogy with the dynamical model shown in Fig. 8 of Part I suggests that further north in the N. Pacific there may actually be a southward directed western boundary current. South of latitude 55°S the pattern of flow seems to depend rather critically on what transport is permitted to flow through Drake Passage between the southern tip of South America and the northern tip of the Palmer Peninsula on the Antarctic Continent. It has been pointed out, however, in an earlier article (STOMMEL, 1957) that Drake Passage does not afford a free zonal passage for deep flows because of the position of the island arc on the eastern side. All except surface layers must (and indeed do) move northward in passing through. We do not have an adequate understanding of just what happens dynamically in this region, and as yet cannot introduce this complication theoretically in a quantitative manner. Therefore, in later sections the flow through

Drake Passage will have to be carried as an undetermined parameter. In Fig. 4 the rather drastic assumption that it vanishes has been made.

ESTIMATE OF THE AMPLITUDE OF THE ABYSSAL CIRCULATION ON THE BASIS OF TRADITIONAL OCEANOGRAPHIC INFORMATION

We have constructed in Fig. 4 a schema of the abyssal circulation, showing the patterns which would ensue if there is a general upward motion over the entire ocean at mid depths, fed by two equal point sources in the North and South Atlantic. We have not assigned an amplitude to this flow pattern. How much water is actually flowing in this pattern, in millions of cubic metres per second ? We now proceed to attempt to answer this question, first by comparison with the quantities of flow observed directly in the March – April 1957 *Discovery II – Atlantis* Expedition to the western boundary current in the North Atlantic, and secondly by a study and simplification of the dynamic calculations of the data recently published by WÜST (1955 and 1957). In the next section we shall determine the amplitude of the abyssal circulation by means of the new theory of the oceanic thermocline of ROBINSON and STOMMEL (1959).

We first consider the results of the joint British-American expedition to survey the deep currents under the Gulf Stream off Charleston, North Carolina, U.S.A., in the Spring of 1957. To date, only a short notice of the results has been published (SWALLOW and WORTHINGTON, 1957) and it is possible that estimates of deep flow will be rendered somewhat differently in the forthcoming more deliberate account of the survey which these authors are preparing, but if the depth of no axial motion in the western North Atlantic is about 1500 m., geostrophic calculations made on sections from Nova-Scotia to Bermuda and from Chesapeake Bay to Bermuda indicate that there are about 20×10^6 m^3/sec flowing southward in a narrow intense western boundary current along the coast of the United States. (However, when dynamic calculations of transport in the undercurrent off Charleston were made by WORTHINGTON and SWALLOW – as yet unpublished – much lower values were obtained : 3 to 6×10^6 m^3/sec). The transports computed by WÜST (1957) from the South Atlantic *Meteor* data show that the North Atlantic Deep Water flows southward along the Brazilian coast – riding over an intruding wedge of Antarctic Bottom Water – with about the same transport. Of course there is an arbitrary element in all dynamic current calculations based on hydrographic data alone – namely the choice of reference level – but the water mass characteristics in the South Atlantic are so very clear-cut that we are inclined to place confidence in WÜST's choice of reference level. We identify this narrow current of North Atlantic water with the western boundary current in our theoretical schema (Fig. 4) and think of it as merely being a somewhat attenuated continuation of the one under the Gulf Stream. Unfortunately we do not have reliable bases for estimating the deep flows in other oceanic areas, and so we cannot press this method further. From the T-S relations of deep water for the whole world it looks as though the contribution of flow from the Weddell Sea must be at least as much as that from the North Atlantic – at least another 20×10^6 m^{3}sec^{-1} so that the total rate of addition of water from the two sources S_0 and S_1 in Fig. 4 must be reckoned as at least 40×10^6 m^3/sec.

Having made reasonable guesses of the values for the two sources, we can now attach numerical values to all other parts of the circulation, and in so doing we naturally must make a crude allowance for the various dimensions of different oceanic basins. In Fig. 5 we have divided the world ocean into 14 portions :

North Atlantic into two sub-areas each of 2 unit areas

South Atlantic	2	2
North Indian	2	1
South Indian	2	3
North Pacific	2	5
South Pacific	2	5
Southern Ocean south of Africa	1	2
Southern Ocean south of Australia	1	2

The total area of the world ocean is thus divided into 40 unit areas. Inasmuch as the actual area of the 3000 m. surface in the world ocean is about 3×10^8 km^2, we must associate $7{\cdot}5 \times 10^6$ km^2 with each of the above unit areas. Although the

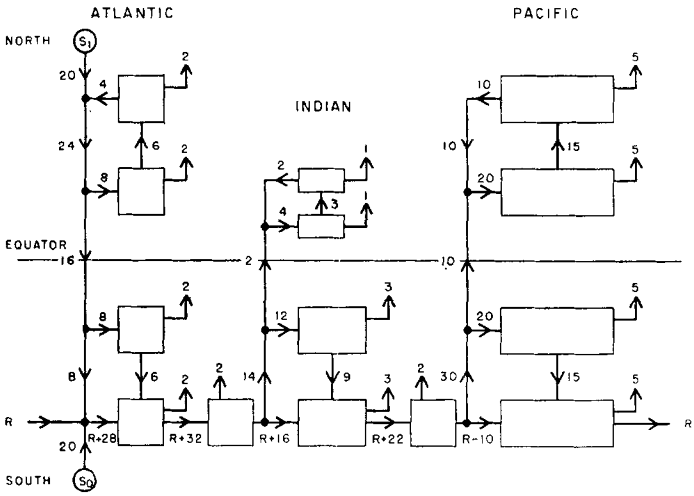

FIG. 5. Schematic budget of transports in various portions of the World Ocean (see text).

theory of the oceanic thermocline (ROBINSON and STOMMEL, 1959) indicates that the vertical velocities at mid-depths are functions of position we will, for simplicity, assume that the vertical velocity is nevertheless uniform over the ocean, in which case 1×10^6 m^3/sec flows upward through each unit area. The upward-bent arrows at the upper right hand corner of each box in Fig. 5 represent this upwelling. The figure associated with these arrows is the approximate total upward flux across the 3000 metre level out of each box. The meridionally directed arrows connecting boxes (such as the one labelled 15 between the two Northern Pacific sub-areas) is meant to represent the interior flow from one sub-area to another. The flow (in 10^6 m^3/sec) between the western boundary currents and each box is indicated by a zonally directed arrow on the left of the box. Thus there is a flux from the western boundary current to the southern sub-area in the Northern Pacific of 20×10^6 m^3/sec and a flow from the northern sub-area of the North Pacific into the western boundary current of 10×10^6 m^3/sec. As we can see, only half of the water entering the interior of the North Pacific actually flows directly up through the thermocline, the other half recirculates through the western boundary current in accordance with the theory of Part I, section 3. The total flux of 10×10^6 m^3/sec up through the main thermocline in the Northern Pacific is supplied by a western boundary current across the equator, there being no mean flow across the equator at any distance from the western boundary. A similar picture prevails in the North Indian Ocean, but on a much smaller scale, the areas involved being so much smaller than those of the North Pacific. In the North Atlantic the interior flow patterns are much the same, but the strength of the western boundary current is much increased and flows toward the south and across the equator, the recirculation flow being amplified by one of the major deep water sources, S_1. The northern sub-areas of the major southern basins are similar to those just above the equator, but the flow pattern in the southernmost sub-areas (the bottom five in Fig. 5) are quite different. The flow through Drake Passage is not determined in our model so that we must assign it an arbitrary value R, which stands for another kind of recirculation, the amount of water which goes round and round the Antarctic Continent through Drake Passage. The rest of the numbers are obtained from simple continuity requirements. One of the most extraordinary features of this construction is the large flux (30×10^6 m^3/sec) in the western boundary current in the South Pacific, located we suppose, somewhere in the vicinity of the Tonga–Kermadec Trench. Other features of interest, though not so marked, are the reversals in directions of the deep western boundary currents in the Northern Pacific and Indian Oceans. Another feature, which we have also mentioned before (STOMMEL, 1958) is the fact that the cold deep waters flow away from the equator in the interior of the oceans.

Having described Fig. 5, let us now pause to interpret its meaning, and to review its implications. First, we have paid no attention to the probable disturbing effects of pronounced bottom topography within ocean basins. This will doubtless cause deviations from the extremely simple schema we have presented. However, it does not seem worthwhile to try to be more sophisticated than the present stage of our understanding and observational knowledge of the abyssal circulation demands. Similarly, since our main purpose is to imitate the grossest features of the abyssal circulation, we do not pursue the refinements of geographical variation of the upward flux of water to which the theory of the oceanic thermocline would naturally lead us: we simply take a constant and uniform upward velocity over the whole ocean. The pattern of flow in the interior of the deep ocean is thus fixed. Its amplitude remains to be determined. The flow in the interior is independent of the positions of the sources (S_0 and S_1). In a steady state the sum of the sources must be equal to the vertical flux over the entire ocean. Wherever the sources may be we can connect them to the interior solutions as necessary by a system of western boundary currents and the special zonal currents that connect western boundary currents at different boundaries.

We envisage the ultimate cause of the abyssal circulation to be the mechanism of the main thermocline which in effect, as a result of surface heating and applied wind stress, demands a certain upward flux at mid-depth. In the next section we will compute this upward flux from the theory of the oceanic thermocline (ROBINSON and STOMMEL, 1959).

The exact location of the sources is, according to our view, more or less a climatological accident. One could disappear and another appear somewhere else – for example, in the North Pacific – with changing climatic conditions, but these would be accompanied by no major modifications of the interior region of the abyssal circulation, and with no major change of the amplitude of the overall abyssal circulation. Of course any changes in the parametric mixing in the ocean would have effects on the rate of the abyssal circulation, and if the wind convergences were greatly intensified, this might also reduce the overall amplitude of the abyssal transports. This way of looking at the abyssal circulation is quite different from the traditional view which implies that the magnitude of the abyssal circulation is determined essentially by the amount of winter cooling at the polar regions and hence that a warming of polar regions of only a few degrees would largely stop the abyssal flow. From our point of view a warming of polar regions of one or two degrees would not affect deep transports except possibly to shift the location of the sources, and to reshuffle the western boundary currents. The deep temperatures would eventually rise a few degrees, and the distribution of oxygen might change – especially if sources shift position – but the overall transports, and the depth of the thermocline etc. would remain unchanged, providing the climatological factors which determine the thermocline remain unchanged.

ESTIMATE OF THE AMPLITUDE OF THE ABYSSAL CIRCULATION ON THE BASIS OF THE THEORY OF THE THERMOCLINE

A dynamical model of the thermocline in principle makes it possible to estimate the vertical component of velocity at mid-depths from the observed distribution of temperature (or more properly density) in the ocean. The calculation is only an estimate because there are simplifications and idealization in the theory which at best are only approximately true in nature and because there appear to be irregularities of a more or less short period observed in hydrographic data which make it impossible to obtain a sufficiently precise mean distribution of properties (in present day language, ‘ there is a high noise level ’), so that one can obtain only a rough estimate of various derivatives of the temperature and salinity fields.

The calculation may be made in a variety of ways.

(1) Pairs of hydrographic stations at the same latitude may be chosen, and a calculation of the depth of no meridional motion, and of the vertical component of velocity, made by the numerical integration of the linearized vorticity equation (STOMMEL 1956, equation 8). To make sure of this method it is necessary to have a good estimate of the mean curve of the wind-stress at the surface, to which unfortunately the results are quite sensitive. If the hydrographic station data contain short period variabilities (internal waves) the calculations of horizontal gradients of density can be made only very crudely. This difficulty has been encountered by most oceanographers employing ‘ dynamic calculations.’ In order to obtain even a fair representation of the currents the pairs of stations must be chosen fairly far apart, or else a line of stations must be smoothed in order to obtain a mean horizontal gradient. The method has one major advantage – it does not require hypothetical statements about relative magnitude of terms in the equation of density transport. Due to the uncertainties introduced through the hydrographic data and by the estimates of

curve of wind-stress the numerical results are doubtful – and it is impossible to state definitely even the range of doubt. STOMMEL's 1956 calculations probably yield vertical velocities that are too high by a factor of ten.

(2) Alternatively, one may make a comparison with an idealized theoretical model such as that of ROBINSON and STOMMEL.

If we start with the same assumptions that ROBINSON and STOMMEL (1959) employ in their theory of the oceanic thermocline, except that we use spherical co-ordinates instead of a beta-plane, we have the following set of physical equations. First the dynamical equations are simplified (see Part I) by the traditional approximations used in spherical co-ordinates for a shallow fluid layer on a rotating globe to the linear geostrophic set :

$$- z\omega a \cos\theta \cdot u = - \partial p/\partial\theta \tag{4·1}$$

$$- z\omega a \cos\theta \cdot v = - \partial p/\partial\phi \tag{4.2}$$

$$g(1 - \alpha T) = - \partial p/\partial z \tag{4.3}$$

The equation of continuity is :

$$a \sin\theta \frac{\partial w}{\partial z} + \frac{\partial}{\partial\theta}(u \sin\theta) + \frac{\partial v}{\partial\phi} = 0 \tag{4.4}$$

and the equation for the temperature transfer is :

$$w \frac{\partial T}{\partial z} + \frac{u}{a}\frac{\partial T}{\partial\theta} + \frac{v}{a\sin\theta}\frac{\partial T}{\partial\phi} = \kappa \frac{\partial^2 T}{\partial z^2} \tag{4.5}$$

where T is temperature and κ is the parametric mixing coefficient which is discussed at length by FOBINSON and STOMMEL. Equations (4.1) to (4.5) lead to the following set of equations in u, w and T, which in fact are of the same form as the corresponding set in the beta-plane :

$$\frac{\partial^2 w}{\partial z^2} = \left[\frac{g\alpha}{2\omega a^2 \cos^2\theta}\right]\frac{\partial T}{\partial\phi} \tag{4.6}$$

$$\frac{u}{a}\frac{\partial T}{\partial\theta} + w\frac{\partial T}{\partial z} = \kappa\frac{\partial^2 T}{\partial z^2} \tag{4.7}$$

Hence there is no difficulty in immediately applying the same similarity transformation as before. In applying the theory to the real ocean the actual surface temperature cannot be used unless full account is also taken of the wind-stress distribution as well. However, if the origin ($z = 0$) is taken at the level where $w = 0$ – that is roughly at the inflexion point in the temperature sounding – the temperature boundary condition may be taken as

$$T = T_0 \sin^n\theta$$

where of course T_0 is not the amplitude of observed temperature range at the surface, but at the centre of the main thermocline. We obtain the following expression for the vertical component of velocity just beneath the thermocline

$$w(-\infty) = T_0 \sin^n\theta \left[\frac{4}{3}\frac{g\kappa^2}{2\omega T_0^2}\frac{\alpha}{a^2\phi}\frac{\sin^{1-2n}\theta}{\cos^2\theta}\right]^{\frac{1}{3}} \tag{4.8}$$

If we could neglect the influence of the wind we would probably set $n = 3$, which would give a pretty good fit to the observed surface temperature distribution in the oceans. However, as shown in ROBINSON and STOMMEL (1959), extensive regions of Ekman convergence produced by the wind at the surface force the level of no vertical component of velocity several hundred metres beneath the surface, and thereby reduce the amplitude of the temperature which should appear in the above expression for the vertical velocity to about one-half the surface amplitude. Instead of $T_0 = 30$ °C we should use $T_0 = 15°$ as an approximation. This has very little influence on the thickness of the thermocline – since it affects it only by a multiplicative factor of the cube root of two, but the amplitude of the vertical component of velocity is directly proportional to this temperature amplitude so that it is reduced to a half of what it would be in the absence of the wind – other factors being equal.

We now wish to perform an integration of the above expression for the vertical component of velocity at mid-depth, over the surface of a single hemisphere globe completely covered with water and bounded by a complete meridian. This is not a bad approximation to the area of the 3000 m. level on the real ocean (about 60 per cent in fact). The limits of integration therefore will be

$$\theta = o, \qquad \theta = \pi/2, \qquad \phi = o, \qquad \phi = 2\pi$$

and the total integral I is :

$$I = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} w\,(-\infty)\, a^2 \sin\theta \, d\theta \, d\phi$$

or

$$\doteqdot 11 \left[\frac{g\alpha T_0}{\omega} a^4 \kappa^2\right]^{\frac{1}{3}} \tag{4.9}$$

Now setting in the following numerical values used by ROBINSON and STOMMEL :

$$g = 10^3 \text{ cm sec}^{-2} \qquad \alpha = 2 \times 10^{-4} \, °\text{C}^{-1}$$

$$\omega = 0{\cdot}7 \times 10^{-4} \text{ sec}^{-1} \qquad \kappa = 0{\cdot}7 \text{ cm}^2 \text{ sec}^{-1}$$

$$a = 0{\cdot}64 \times 10^9 \text{ cm} \qquad T_0 = 15 \, °\text{C}$$

we obtain

$$I = 50 \times 10^6 \text{m}^3\text{sec}^{-1}$$

which is in reasonable agreement with the minimum value of $40 \times 10^6 \text{m}^3/\text{sec}$ obtained in the previous section from estimates of flow from the cold sources S_0 and S_1. The average amplitude of the maximum vertical velocity is 4×10^{-5} cm/sec. Since the maximum upward velocity component is at a depth of about 1000 m. on the average, and there is on the average about 3000 m. of water beneath this depth, the deep water is replaced (in a very gross sense of course we have to allow for particular deviations) every 200 years. Although a calculation of this kind is simple because it uses a minimum of data – all the integrations are performed before comparison of the model with observation – it is rather misleading in its simplicity because it contains two implicit assumptions : Namely, that the eddy mixing parameter does not vary geographically (or with depth) and that horizontal mixing and the east-west component of density advection are negligible. These assumptions cannot be fully justified, and the numerical values of amplitude of the abyssal circulation obtained in the manner outlined above are thus subject to serious uncertainty.

(3) An intermediate approach is therefore indicated. Observational material is used to give an indication of the thickness of the thermocline at a number of different latitudes in the western sides of the interior of each ocean basin. The ROBINSON–STOMMEL theory for infinite depth model is used to compute the deep vertical component of velocity. If there are geographical variations in the mixing parameter they are thus taken into account (the hypotheses that the vertical eddy coefficient is constant in depth, and that horizontal mixing east-west advection are negligible, remain). In order to avoid calculation of too many quantities, zonal averages in each ocean can be calculated by direct x – integration using the analytical form introduced by the similarity transformation. The final summation is a simple numerical integration, where each zonal average of vertical component of velocity is multiplied by an appropriate area, so as to cover the whole ocean; the final sum is the total amplitude of the abyssal circulation. Considering the influence of errors in procedure (1) and of unsubstantiable hypothetical elements in the procedure (2), it seems to us that this intermediate procedure (3) is more likely to give good results, although one can hardly expect an accuracy of better than a factor of two or three.

Thus, using the notation and equation number of ROBINSON and STOMMEL (1959) we want to compute the average of w_∞ – the deep water asymptotic value of the vertical component of velocity along a latitude circle from one side to the other of the interior of a real ocean basin. One of the quantities which we want to avoid having to estimate is w_e, the vertical velocity just beneath the Ekman layer. Between equations (35) and (37) (of ROBINSON and STOMMEL 1959) the similarity transformation W_e of w_e can be eliminated to obtain the following form :

$$W_\infty = -\frac{L^2\,\theta_1}{4}$$

where θ_1, is the temperature near the middle of the thermocline, and L is a transformation of thermocline thickness z_t.

$$L = 2\,(\epsilon/x)^{\frac{1}{3}}\, z_t$$

where ϵ is the dimensionless small ratio $(\alpha\beta g)/(3f^2(y)$

Also $$W_\infty = (x/\epsilon)^{\frac{1}{3}}\, w_\infty$$

The calculations for which these formulae hold include the effect of wind-stress at the surface.

The thickness of the thermocline, z_t, is a function of distance from the eastern coast, $-x$ and for a position far in the western side of the interior, $x = x_0$, is measured as $z_t\,(x_0)$. The quantity $z_t\,(x_0)$ is the vertical distance (in centimetres) over which the temperature decreases exponentially to e^{-1} of the mid-thermocline value. By eliminating W_∞ and L, the following expression is obtained :

$$w_\infty = -\,\epsilon\,[z_t\,(x_0)]^2\,(x - x_0)^{-\frac{1}{3}}\,\theta_1$$

The average of vertical velocity taken along a latitude circle between $x = 0$ and $x = x_0$ is :

$$\overline{w_\infty} = \frac{1}{x_0}\int_{x_0}^{0} w_\infty\, dx$$

The theoretical model uses the condition $z_t = 0$ at $x = 0$ as an eastern boundary condition. In the real ocean z_t is smaller is $x = 0$ than elsewhere but does not actually vanish.

$$w_\infty = \frac{3}{2}\frac{1}{x_0}(\epsilon\,[z_t(x_0)]^2\,\theta_1)$$

If the depth of the thermocline does not vanish at $x = 0$, the factor 3/2 is replaced by unity.

For the North Pacific Ocean we might take very roughly the calculation as shown in Table 1. Evidently the zonal average of subthermoclinal vertical component of velocity cannot be calculated nearer to the equator than about 10° by the model

Table 1

Lat °*N*	$z_t(x_0)$ (cm)	ϵ (°C^{-1} cm^{-1})	θ_1 (°C)	$\theta_1\,\epsilon z^2$	x_0 (cm)	w_∞ (cm sec^{-1})
60	6×10^4	$0{\cdot}5 \times 10^{-6}$	5·0	$9{\cdot}0 \times 10^3$	4×10^8	$2{\cdot}5 \times 10^{-6}$
50	5	0·8	5·0	10·0	6	2·0
40	4	1·4	10·0	22·0	8	3·0
30	4	2·4	10·0	38·0	10	3·8
20	3	6·0	10·0	54·0	10	5·4
10	2	24·0	10·0	96·0	10	9·6
0	2	∞	10·0	∞	10	∞

Table 2

METHOD	*Total flux in world abyssal circulation* (10^6 m^3 sec^{-1})	*Vertical velocity under thermocline* (cm day^{-1})	*Replacement time of deep water* (years)
T-S diagram to establish proportions of North and South Atlantic water present; and direct deep current measurements off Charleston to give flux of North Atlantic component	15	0·5	1800
Same T-S analysis and WORTHINGTON's Nova Scotia section and 2000 metre reference level to establish flux of North Atlantic Component	40	1·3	650
Same T-S analysis and WÜST's dynamic calculations for South Atlantic Ocean for flux of North Atlantic component	50	1·6	550
Depth of the thermocline over the world ocean and theory of thermocline to establish total flux	90	3·0	300

proposed by ROBINSON and STOMMEL. This is obvious because the parameter ϵ is not small for latitudes less than 10° and because such observed features of the real ocean circulation as the Cromwell Current (KNAUSS 1959) are not shown when the theory is applied is this inadmissable range. If we can assume that the deep vertical velocity at the equator is not in some way quite singular we can compute averages and obtain thus a figure of abour 4×10^{-5} cm sec^{-1} as a North Pacific average (excluding a band of latitude south of 10 °N). This figure may be off by a factor of three, but at least the estimate of the mean wind-stress and the errors due to internal waves and details of hydrographic structure do not enter.

In a letter to the editor of *Deep-Sea Research*, STOMMEL (1958) gave a value of 3×10^{-5} cm sec^{-1} based on a much cruder thermo-convective model. Both of these are perhaps in fact too large by a factor of two or three. A figure more nearly in accord with WÜST's (1957) geostrophic calculation is $1{\cdot}5 \times 10^{-5}$ cm sec^{-1}. A smaller value than this is obtained if we assume that the transport in the deep western boundary current in the North Atlantic is always the same as that measured in March–April 1957 by the joint British–American expedition (SWALLOW – WORTHINGTON 1957) 6×10^{12} cm^3/sec. Since the T-S properties of abyssal water (Section 3) indicate that the total flux of the abyssal circulation is from 2 to 3 times that of the western boundary current in the N Atlantic, we might choose 14×10^{12} cm^3/sec as the appropriate figure. To obtain the world average vertical velocity we divide this figure by the area of the ocean at 2000 m. (3×10^{18} cm^2) and obtain a figure of: $0{\cdot}5 \times 10^{-5}$ cm sec^{-1}. Table 2 summarizes these calculations.

EFFECT OF LATERAL MIXING ON THE DYNAMICS OF FLOW

In the preceding sections we have proposed a very highly idealized model of the abyssal circulation. The theoretically predicted flow patterns are based on the fundamental assumptions that (a) flow in the interior of the ocean is geostrophically balanced with negligible *dynamical* influence from frictional or eddy effects (b) significant departure from geostrophy occurs only in boundary currents along the edges of continental masses. Since it is clear that both lateral and vertical eddy diffusivity play a very important role in many oceanic processes and in the distribution of tracer properties of the water masses, it is important to inquire, at least in terms of orders of magnitude, as to whether Austausch coefficients large enough to be important in determining the distribution of oxygen, radio carbon, salinity, etc. are at the same time large enough to affect the dynamics of the postulated planetary motions driven by source-sink distributions.

To answer this question it is appropriate to assess the order of magnitude of the various terms appearing in the dynamical equations and in the diffusion equation.

Consider the dynamical problem in the form expressed by the vorticity equation for steady two-dimensional flow on a β-plane, retaining lateral friction terms :

$$u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} + \eta\,\mathrm{div}_H\,\mathbf{v} + f\,\mathrm{div}_H\,\mathbf{v} + \beta v = A_H\left[\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right]$$

In this notation :

x, y co-ordinates positive toward the east and north respectively,

u, v particle velocities corresponding to co-ordinates x, y respectively.

$\mathbf{v}$, total vector velocity

$$\eta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

div_H, horizontal divergence

f, Coriolis parameter

β, $\partial f/\partial y$

A_H, horizontal (lateral) eddy coefficient.

Transforming Equations 3·1 of Part I to the β-plane as defined above, the zonal and meridional components of geostrophically balanced flow for a uniformly distributed sink Q_0 over the entire surface are :

$$u = 2Q_0 (x_2 - x)/h \tag{2.2}$$

$$v = fQ_0/\beta h \tag{2.3}$$

$$\eta \doteqdot - \frac{2Q_0 (x_2 - x)}{h \quad a} = - \frac{u}{a} \tag{2.4}$$

The order of magnitude of Q_0, estimated in section 4 is 4×10^{-5} cm/sec. We adopt the following values for various relevant quantities :

Horizontal distance scale : $X \sim Y \sim 6000$ km $= 6 \times 10^8$ cm

Thickness (depth) of abyssal layer : $h \sim 3$ km $= 3 \times 10^5$ cm

Coriolis parameter : $f \sim 10^{-4}$ sec^{-1}

$\beta \sim 2 \times 10^{-13}$ sec^{-1} cm^{-1}

Radius of the earth : $\sim 6{\cdot}5 \times 10^8$ cm

Vertical velocity at top of abyssal layer : $Q_0 \sim 4 \times 10^{-5}$cm/sec

From these assumed values we estimate the order of magnitude of the following :

$$u \sim 0{\cdot}16 \text{ cm/sec}$$

$$v \sim 0{\cdot}067 \text{ cm/sec}$$

$$\eta \sim + 2{\cdot}5 \times 10^{-10} \text{ sec}^{-1}$$

$$\text{div}_H \mathbf{v} \sim - 1{\cdot}3 \times 10^{-10} \text{ sec}^{-1}$$

$$\partial\eta/\partial x \sim - 4{\cdot}1 \times 10^{-19} \text{ sec}^{-1} \text{ cm}^{-1}$$

$$\partial\eta/\partial y \sim + 3{\cdot}8 \times 10^{-19} \text{ sec}^{3-1} \text{ cm}^{-1}$$

$$\frac{\partial^2 \eta}{\partial x} \sim \frac{\partial^2 \eta}{\partial y^2} \sim 7 \times 10^{-28} \text{ sec}^{-1} \text{ cm}^{-2}$$

(The magnitudes of the derivatives have been estimated by finding the appropriate derivates of Equations 3.1, Part I and then converting to the β-plane).

Thus we have the following orders of magnitude for the terms in Equation (2.1).

$$u \frac{\partial\eta}{\partial x} \sim - 7 \times 10^{-2}{}_0 \text{ sec}^{-2}$$

$$v \frac{\partial\eta}{\partial y} \sim 3 \times 10^{-20} \text{ sec}^{-2}$$

$$\eta \, \text{div}_H \mathbf{v} \sim - 3 \times 10^{-20} \text{ sec}^{-2}$$

$$f\,\mathrm{div_H}\,\mathbf{v} \sim -\,1{\cdot}3 \times 10^{-14}\,\mathrm{sec}^{-2}$$

$$\beta v \sim +\,1{\cdot}3 \times 10^{-11}\,\mathrm{sec}^{-2}$$

It is clear that the dominant terms on the left-hand side of Equation (2.1) are $f\,\mathrm{div_H}\mathbf{v}$ and βv. It is, of course, the balance of these terms which represents the situation we describe as 'purely geostrophic flow.' We can now find values of the Austausch coefficient A_H which would make the right hand side of the equation assume magnitudes comparable with the dominant terms on the left. Noting that the latter are of the order of 10^{-14} and that the second derivatives on the right are of the order of 10^{-27}, it is apparent that A_H, must be of the order of 10^{12} cm²/sec before the frictional terms on the right attain the order of one tenth of the planetary divergence terms.

Available estimates of the large scale lateral eddy coefficient for the ocean usually fall in the range 10^6 to 10^8 cm²/sec. In the light of these results we feel that our fundamental assumption, that the very low velocity flows induced in the interior of the ocean are essentially geostrophically balanced, is a reasonable one. We not do expect lateral friction to affect seriously the purely geostrophic dynamical foundation of the basic circulation patterns. (It is interesting to note that BURGER (1958) obtains a similar result for large scale flows in his consideration of planetary motions in the atmosphere).

We now make a similar scale and order of magnitude analysis of diffusion of a tracer property of the water. The basic differential equation is of the form :

$$A_H\,\nabla^2{}_H\,c - \mathbf{v}\cdot\nabla_H\,c - \lambda\,c = 0 \tag{2.5}$$

where c denotes concentration of a property (say oxygen) and λ is the reciprocal of the " half life " of decay. The three terms of Equation (2.5) represent a balance among diffusive transport, advective transport, and decay, the amplitude of the advective term being determined by the basic geostrophic flow.

To examine the relative significance at the planetary scale of the three terms in Equation (2.5) we re-write it in the following 'operator' form :

$$\left[A_H\,\nabla^2{}_H - \mathbf{v}\cdot\nabla_H - \frac{3{\cdot}17 \times 10^{-8}}{T}\right] c = 0 \tag{2.6}$$

where T is the half-life of decay in years of the tracer property. Introducing the scale magnitudes used earlier, the orders of magnitude of the terms appearing in (2.6) are indicated by :

$$\left[2{\cdot}8\,(10^{-18})\,A_H - 2{\cdot}7\,(10^{-10}) - \frac{3{\cdot}17 \times 10^{-8}}{T}\right] c = 0 \tag{2.7}$$

All three terms are of roughly comparable significance when A_H is of the order of $10^7 - 10^8$ cm²/sec and T is of the order of $10^2 - 10^3$ years. If the half life becomes much less than 10^2 years, the distribution of the property is determined principally by diffusion in the immediate neighbourhood of the western boundary current, advective effects are insignificant, and appreciable concentrations do not extend very far into the interior of the ocean. If T and A_H are both large, the property becomes essentially uniformly distributed, the advective effects insignificant, and the concentration level determined by the relation between half life and rate of supply.

The amplitude of the basic geostrophic circulation in the model we have postulated is such that Austausch coefficients of the order of $10^7 - 10^8$ cm^2/sec will have a very marked diffusive effect on the distribution of tracer properties of the water masses. At the same time, as shown earlier in this section, coefficients of this magnitude would be expected to have a *negligible* effect on the dynamics of the basic flow. This rather paradoxical result arises from the very large scale and low velocity of the basic flow, the frictional force terms being proportional to third spatial derivatives of the geostrophic velocities. It is felt therefore that the fundamental model, in which an interior, large scale, geostrophically balanced circulation is driven by the mechanism of the thermocline, is basically reasonable. It is simultaneously to be pointed out that great caution must be exercised in drawing inferences about the basic flow from observed distributions of water mass properties; the latter probably being seriously affected by diffusion on a scale substantially smaller than that of the basic flow.

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