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Satellite-Sized Planetesimals and Lunar Origin

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Exploratory calculations using accretionary theory are made to demonstrate plausible sizes of second-largest, third-largest, etc., bodies at the close of planet formation in heliocentric orbits near the planets, assuming asteroid-like size distributions at the start of the calculation. Many satellite-sized bodies are found to be available for capture, cratering, or collisional fragmentation. In the case of Earth-sized planets, the models suggest second-largest bodies of 500 to 3000 km radius, and tens of bodies larger than 100 km radius. Many of these interact with the planet before suffering any fragmentation events with each other. Collision of a large body with Earth could eject iron-deficient crust and upper mantle material, forming a cloud of refractory, volatile-poor dust that could form the Moon. Other satellite systems may have been affected by major capture or collision events of chance character.

I. INTRODUCTION

The state of interplanetary matter at the time the planets reached their present masses is important from two points of view. First any theory of such matter can be tested against the sizes and distributions of craters and large multiringed basins which have now been observed on seven planetary bodies in the solar system. These craters and basins must represent the sweep-up of the final groups of planetesimals at the close of planet formation. Second, some satellites are suspected of being captured planetesimal bodies. A theory of interplanetary matter at the close of planet formation is needed to examine the plausibility of satellite capture hypotheses.

The Earth–Moon system is a special case in point. Long ago it was hypothesized that the Moon might have been captured by the Earth but this theory has often been dismissed on the grounds that a capture event is very improbable. Upon the other hand, as emphasized by Urey (1952, 1972), if the early solar system was densely populated with lunar-sized bodies, then such a capture event is more probable. Hartmann (1972) pointed to the extremely high early lunar cratering rate as evidence for many large planetesimals.

In this paper we attempt numerical reconstruction of plausible size distributions of the bodies of second largest size, third largest size, and so on, near the planets at the close of planet formation. These calculations assume that the planets grew primarily by accretion of small particles onto initial larger bodies. The accretionary processes are compatible with the models outlined by Alfven and Arrhenius (1970a, b). The approach used also derives from an accretionary model by Hartmann (1968, 1970), which (in part) views the largest asteroids as those that had just begun to accrete gravitationally and were left in their present state when the solar nebula dissipated. This work follows in turn from earlier suggestions by Anders (1965).

Our method here is to apply the ordinary equations of growth in an accreting system, where certain initial size distributions are specified. Assumed initial sizes are based in part on the present sizes found among the larger asteroids, since we know that in at least one case a group of planetesimals reached this state. The fraction of collisions in which pairs of particles stick together can be assumed to be a constant whose value affects only the time-scale processes. According to arguments such as those given by Hartmann (1971), derived from Öpik's (1963) work, the approach velocities among the planetesimals can be anticipated to increase slowly during the accretion process due to mutual perturbations and dissipation of a resisting medium. Here we have allowed for various plausible but ad hoc evolutions of the relative approach velocities of the particles. The approach velocity can be seen to be very important to the time-scale because the slower the approach velocity, the larger the effective gravitational cross section of the planetesimal. If we consider a growing particle, it initially sweeps out a volume equivalent only to its geometric cross section. However as it grows larger it eventually reaches a *critical radius* R_c at which its gravitational field and its capture cross section begin to increase as r^4 instead of r^2. Clearly this lets such a particle begin to sweep-up material at a much faster rate than its neighboring smaller particles. It is for this reason that our results show a general departure of the larger particles sizes from the sizes of the smaller companions. It is for this reason, too, that planets can be expected to reach nearly their present dimensions surrounded by a swarm of smaller, but significant, coorbiting particles in heliocentric orbits. In evaluating origins of satellites as well as final stages of cratering history, it is important to estimate the nature of such particles.

II. Accretion Models

The rate of growth of a body accreting material is given by (Alfven and Arrhenius, 1970a, b; Hartmann, 1968, 1970; and others).

$$\frac{dR}{dt} = \frac{V_\infty f \rho_a}{4\rho_p}\left[1 + \left(\frac{R}{R_c}\right)^2\right], \tag{1}$$

where R is the radius of the body, V_∞ is the relative speed of the two bodies far from each other, ρ_a is the space density of accretable material (gm cm^{-3}), ρ_p is the density of the accreting planetesimal (gm cm^{-3}), f is the fraction of material that adheres to the body during a collision, and R_c is the so-called critical radius and is given by

$$R_c = V_\infty\left(\frac{3}{8\pi G\rho_p}\right)^{1/2},$$

G being the gravitational constant (Alfven and Arrhenius, 1970a, b; Hartmann, 1968). This is the radius at which gravitation begins to dominate in (1).

Equation (1) is derived on the basis of a Keplerian trajectory model. If the orbit of the smaller body intersects the surface of the larger body then it is assumed that the smaller body is accreted. The small body is assumed to be a point mass.

Equation (1) exhibits two distinct regimes depending on whether $R \ll R_c$ or $R \gg R_c$. For the radius small compared with the critical radius, then the term $(R/R_c)^2 \ll 1$ and (1) may be approximated by

$$\frac{dR}{dt} \simeq \frac{V_\infty f \rho_a}{4\rho_p}, \tag{2}$$

which gives a growth rate independent of the size of the body itself. However if $R \gg R_c$,then

$$\frac{dR}{dt} \approx \frac{V_\infty f \rho_a}{4\rho_p}\frac{R^2}{R_c}$$

and the growth rate is now greatly enhanced by the large factor $(R/R_c)^2$. This condition occurs when the gravitational field of the planet dominates the accretion process.

In the study, Eq. (1) was integrated numerically for 10 bodies simultaneously. Several assumptions were made in order to perform the integrations. The parameters ρ_a, f, and V_∞ are properly functions of R, as is ρ_p to a lesser extent. In this study, the quantity f was set to unity, hence the models assume that all material that impacts the bodies adheres to them. Treating f as a constant is reasonable in light of the large sizes of the initial masses. Furthermore, V_∞ becomes large as the planetesimals become large, hence any mass wasting due to collisions is minimized. Different nonzero constant f's would affect our time-scales, but not our qualitative conclusions. The functional relation by which V_∞ changes is unknown, but it is believed to increase. After reviewing this problem, we assumed for exploratory purposes and simplicity that V_∞ is linearly increasing with time. The planetesimal density ρ_p was assumed constant throughout the integrations. The nebular density ρ_a varies throughout the integration because the total mass to be accreted into the bodies was specified initially, hence ρ_a decreases as mass is accreted into the planetesimals. The initial value of ρ_a strongly affects the time scale for accretion; however, time scales were of secondary importance for this study. Consequently, ρ_a was initially chosen for plausibility, and such that growth times were on the order of 10^5–10^7 years. Each run, then, required that the V_∞ variation, the total mass and the initial size distribution be specified.

The critical radius, R_c, is proportional to V_∞ for constant ρ_p and Table I lists values of R_c for ρ_p of 3.5 and 5.5 gm cm^{-3}. This table is important in showing the sizes at which rapid planetesimal growth, leading to planets, begins.

A range of V_∞ values, from a few meters per second on the small end to 5 km sec^{-1}, which is representative of the current asteroid belt, was chosen. Two initial size distributions were investigated, drawn from an early semitheoretical model by Anders (1965) and a current listing of large asteroids (Chapman, private communication, August 1974). These are listed in Table II.

Figure 1 illustrates the general shape of three independent growth curves. The figure illustrates growth to 3000 km radius under three different relations between initial radius and critical radius. The critical radius is initially 35 km and its time dependence is also shown. Three initial assumed radii are 1, 35, and 350 km. The bodies grow rather slowly until the critical radius is reached, then display a greatly accelerated growth rate until mass depletion of the initial cloud slows and terminates the growth process. Note that initial radii close to or small compared with the critical radius lead to quite similar growth curves, i.e., the curves for 1 km and 35 km initial radius. However, if the initial radius exceeds the critical radius (350 km curves) then the growth is always quite rapid. Note also that a time-scale of millions of years is required for complete growth from small to planetary dimension, but that the rapid growth occurs in only about 10% of the total time.

Figures 2 and 3 depict the growth of a family of planetesimals having the Anders original size distribution. In Fig. 2 all initial radii are larger than the critical radius. For this case when the largest bodies have grown to Earth's size (6900 km radius), the second largest is 1850 km, the third

TABLE I

Representative Values of the Critical Radius, R_c, as a Function of V_∞

	Critical radius, R_c (km) for:	
V_∞ (km/sec)	$\rho_p =$ 3.5 gm cm^{-3}	$\rho_p =$ 5.5 gm cm^{-3}
0.01	7.1	5.7
0.5	357	285
5	3572	2850

TABLE II

Assumed Initial Planetesimal Size Distributions

	Diameter (km)	
Body	A[a]	B[b]
1	680	1030
2	600	560
3	540	500
4	440	370
5	420	290
6	410	270
7	400	270
8	390	260
9	380	250
10	370	240

[a] Hypothetical initial asteroid distribution (from Anders).
[b] Current largest asteroids.

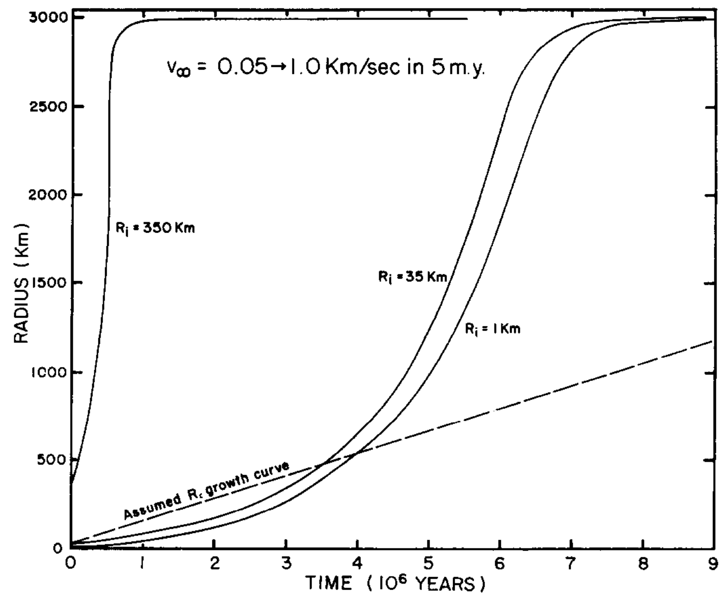

FIG. 1. Three independent growth histories, showing planetesimal radius as a function of time and initial relation to the critical radius, R_c, at which gravitational cross section becomes important. R_i is assumed radius at $t = 0$. Assumed behavior of approach velocity V_∞ is given at top. Rapid growth begins when planetesimal exceeds critical radius. (Each of the three planetesimals was assumed independently to have enough accretable mass available to reach $R = 3000$km.)

largest is 1100 and the tenth body is about 390km. These figures compare to lunar radius of 1734km. In Fig. 3 all the initial radii are small compared with the critical radius. In this case the largest body grows to around 4200, the second largest to 3400km and the tenth body is nearly 2400km in radius. Hence, starting bodies with initial radii small compared with the critical radius leads to a much more uniform distribution of final sizes.

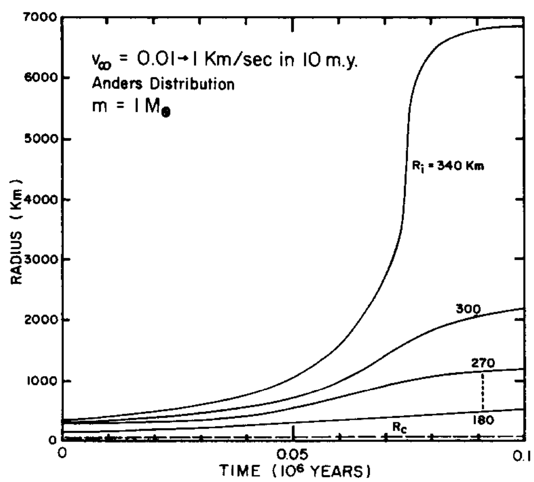

FIG. 2. Growth history for a group of particles having the initial "Anders" size distribution, and embedded in an accretable nebula mass of one Earth mass. Bodies assumed to have already exceeded R_c by $t = 0$.

Figures 4–6 show the results for the initial distribution resembling the known asteroids (Chapman, private communi-

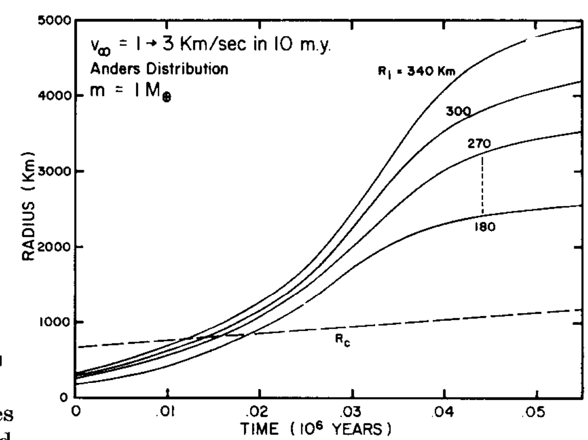

FIG. 3. As in Fig. 2, except assumed V_∞ behavior is different, so that bodies have not yet reached R_c by $t = 0$.

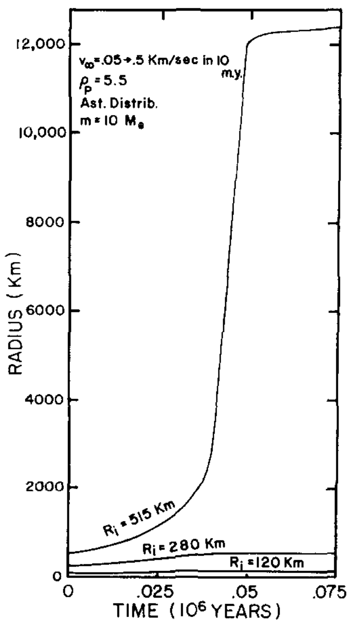

FIG. 4. Growth history for a group of bodies starting with observed asteroidal mass distribution, low V_∞, and low R_c, leading to very rapid dominance of largest body. R_i = radius at $t = 0$.

cation). The dominance of the largest body is due to two factors: (1) the relatively large initial size of the largest body (Ceres), and (2) the initial radii being above the critical radius. In Fig. 6 where V_∞ leads to a large critical radius the final distribution is again relatively uniform. In these figures we have allowed for 10 Earth masses of accretable material in our assumed toroidal volume. In Fig. 4, when the largest body has reached the size of Earth, the second largest has reached only about 550km. Similarly in Fig. 5 the second has reached about 2100km and the third, about 1200km. Finally, under conditions in Fig. 6, the second and third bodies reach about 4800km and 4100km radius by the time the largest reaches the size of the Earth.

These very preliminary calculations suggest that the availability of satellite-sized planetesimals depends strongly on the relative velocities of planetesimals (i.e., critical radius) in the early stages of planet formation. The results also suggest that under reasonable conditions, a number of Moon-sized bodies should have been produced in the vicinity of terrestrial and jovian planets.

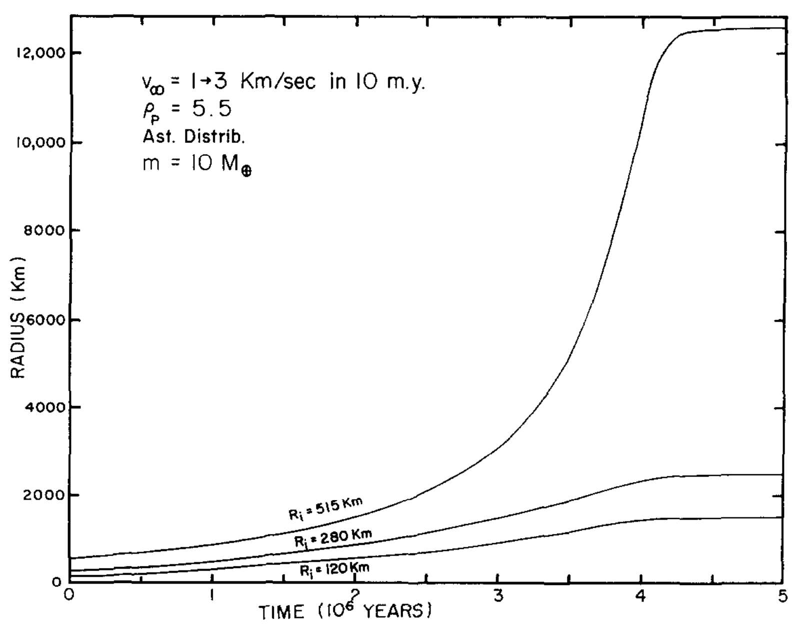

FIG. 5. As in Fig. 4, but with different V_∞ and R_c histories, leading to dominance of planet with substantial secondary bodies.

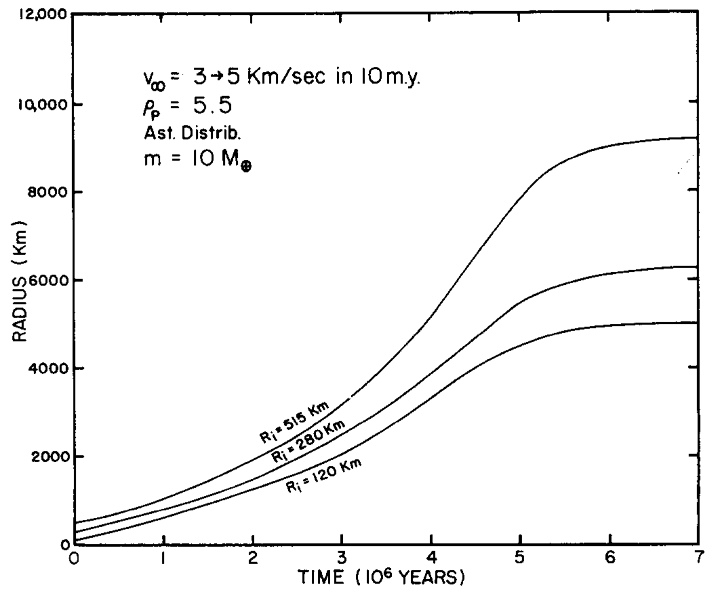

FIG. 6. As in Figs. 4 and 5, but with different V_∞ and R_c evolution, leading to very large secondary and tertiary bodies.

III. COLLISIONAL LIFETIMES

As the bodies are growing via accretion there are other processes occurring which tend to destroy them. Two mechanisms dominate the destruction of these larger bodies (i.e., at least a few hundred kilometers in radius), namely: (1) destructive collisions by smaller bodies which have sufficient energy to disrupt the larger body, or (2) accretion by an even larger body (i.e., the planet). We now estimate the lifetime for each of these processes. For disruptive collisions, the lifetime may be estimated from a "particle in a box" calculation, assuming that the number of smaller bodies capable of fragmenting the large one is known, and that perturbations cause a mixing of particles. From calculations of the energy required to fragment a gravitationally bound body with little internal mechanical strength and assuming a reasonable range of impact speeds and planetesimal properties, we use an estimate of 1/125 the mass of the impacted body as the minimum mass capable of fragmenting a body. This corresponds to a body roughly 1/5 the size of the impacted body. Use of a power law distribution of the form

$$N(m) = Km^{-2/3},$$

where K is a constant, for the number of bodies $N(m)$ having a mass greater than m, enables the collisional lifetime to be estimated, assuming a toroidal "box" with volume $1.6 \times 10^{39}\,\mathrm{cm}^3$. Figure 7 shows the lifetime as a function of the size of the body for two values of V_∞ and for two values of total mass in the accreting cloud. The lifetime is insensitive to the size of the body for $V_\infty = 5\,\mathrm{km\,sec}^{-1}$ and only varies by about a factor of two for the $V_\infty = 1\,\mathrm{km\,sec}^{-1}$. The curves for $V_\infty = 1\,\mathrm{km\,sec}^{-1}$ are truncated at the point where there is insufficient energy in the collision to fragment the body.

If one of the bodies becomes dominant, i.e., approaches planetary size, then the question of the collision with the planet becomes important. This is the second of the processes noted above whereby planetesimals are removed from the swarm. Figure 8 was developed using a similar calculation to the above. It shows the planetesimal lifetimes once the largest body exceeds 4000 km radius. For low V_∞ ($\sim 0.1\,\mathrm{km\,sec}^{-1}$) the lifetime becomes quite short as the body approaches Earth-size and indeed becomes less than 10^6 yr for a 6000 km radius body. For a V_∞ of $1\,\mathrm{km\,sec}^{-1}$ the characteristic lifetime

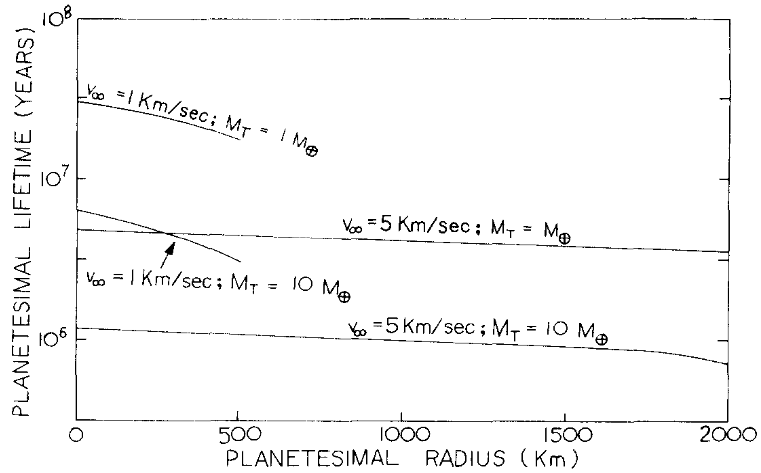

FIG. 7. Estimated half-lives for planetesimals against collisions with smaller bodies large enough to cause fragmentation, based on discussion in Section III. Mass ratio for fragmentation assumed to be 1/125. M_T = total accretable mass in volume considered.

is $\sim 10^7$ yr while for $V_\infty = 5\,\mathrm{km\,sec^{-1}}$ the average lifetime increases to about 6×10^7 yr.

It thus appears probable that once planets form, large planetesimals will fragment each other, collide with the planet (forming basins), or be captured, with a half-life of the order 10^6 to 10^8 yr.

IV. Implications and Future Work

The models developed for this study show that if accretion alone controls the size distribution of bodies at the close of planet formation then many secondary bodies of substantial size relative to the planets may form as well. In fact, under

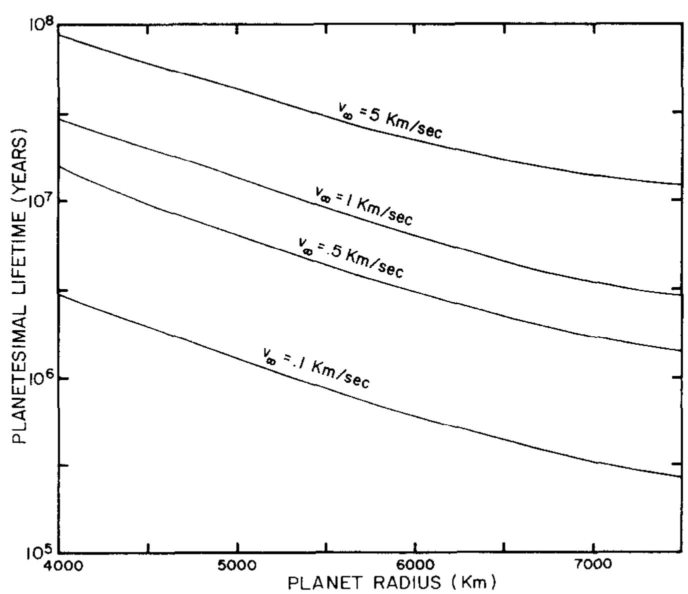

FIG. 8. Estimated half-lives for planetesimals against collision with the planet or other large bodies growing in the toroidal volume discussed in Section III.

some initial conditions the resulting size distribution leads to planetesimals of half the planetary radius or more. A likely size distribution after 10^7 years is one dominant body (planet) and many secondary bodies larger than $R = 100$km. However, the processes of collisional fragmentation and accretion by smaller and larger bodies modify the size distribution relative to that of pure accretion alone; these processes reduce the number and largest size of the intermediate-size bodies. During the course of these processes, the probability of the planet interacting with a large body is much larger than has been considered in some past descriptions of planetary growth. We will consider one specific application of this idea to the Earth in the next section.

The above sequence of models suffers from a lack of knowledge of several key parameters. The one variable whose value affects all these calculations strongly is V_∞. Knowledge of how this parameter varies with time is vital to further development of these models. An improved estimate over the linear increase with time may be obtained from Ruskol (1963) and Safronov (1958, 1972). The mean relative speed is taken as

$$V = (Gm/\theta r)^{1/2},$$

where m and r are the mass and radius of the largest planetesimal and θ is a dimensionless parameter between 1 and 3. This expression leads to V growing linearly with the radius of the largest planetesimal rather than with time. This is intuitively more satisfying, as the largest planetesimal is the dominant source of perturbations on the other bodies. Also the variations of f, the fraction of the impacting body that adheres, need to be incorporated in order to evaluate time-scales and growth at small r more rigorously. This can presently be discussed only with the limited experimental data of Gault *et al.* (1963b) giving the speed distribution of ejecta for cratering events. The value of f depends primarily on the impact speed and the escape speed from the surface of the impacted body.

Baldwin (1974) has counted small craters overlapping larger craters of various ages, and found the very interesting result that premare impactors "contained a much higher proportion of smaller bodies in the earliest observable times than subsequently." Although the size distributions of craters are difficult to interpret in this way, because of effects of deposition of ejecta blankets, secondary impacts, and other events, this result is intriguing in suggesting that growth of planetesimal mean sizes—similar to the growth modeled in this paper—continued even after lunar surface formation. Further work on these crater statistics might help set limits on the growth time-scales and processes to be used in more sophisticated accretion modeling. Also recommended is the development of a simulation that includes collisions and larger body accretion in addition to accretion from the solar cloud of dust particles and small planetesimals.

V. Ramifications for Origin of the Moon

Recent discussions of lunar origin have emphasized several observations including the Moon's (1) lack of iron, (2) depletion in volatiles, and (3) enrichment in refracting elements, relative to Earth and cosmic abundances (Wood, 1975, Table I). Wood (1975) has shown that aspects of all three pre-Apollo theories (capture, fission, binary accretion) are still invoked in various degrees or combinations by various workers. A widely admired theory of Ringwood (1970) contrives to have the Moon condense out of a cloud of hot gas and particles boiled and spun off the Earth by ordinary processes of accretion. Objections include angular momentum and differentiation considerations (Wood, 1975) and the evolutionary nature of the model which might equally predict massive satellites for other planets. Wood (1975) has leaned toward some variant of capture of fission, such as the tidal breakup of a differentiated planetesimal near Earth, with only the low-iron crustal debris remaining in Earth orbit and the rest passing on by Earth into a new heliocentric orbit. This type of near-collision is, of course, ad hox and contrived, but is conceived primarily to produce a cloud of planetary crustal or mantle composition from which the Moon came. Binder (1974) asserts from petrologic studies that the Moon's mineralogy is consistent with formation from such disseminated upper mantle material removed from the Earth. However, the traditional fission models continue to suffer dynamical criticism.

A model of lunar origin having many of the advantages of the above theories, and few of their disadvantages, stems naturally from our work. If a planetary body forms in a certain zone in the solar system, there must be a second-largest body in that zone (and still smaller bodies). Our calculations suggest the probability that some of these bodies have appreciable radii and masses relative to the planet. This is quite consistent with results of Safronov (1966) who models impacts of these bodies to produce obliquities of the planets. Safronov proposes an impact of Uranus with a body of about 0.05 its own mass to produce the obliquity, and Singer (1974) has created a similar model involving the satellites. Similarly, we know from the largest basins on the Moon, Mercury, and Mars, that bodies about 32 to 95km in radius struck the Moon and Mars (Baldwin, 1963) and about 100km in radius struck Mars (Hartmann, 1971), late enough to leave observable scars. (Lunar dates suggest that this was about $5 \ll 10^8$ years after the Moon formed.) Traces of earlier, larger, collisions may have been erased during crustal formation.

Based in part on Fig. 8, we suggest that still larger bodies growing near the Earth's orbit could have struck the Earth within the first 10^7–10^8 years, depending on orbital semimajor axis, about 4.5×10^9 years ago. Half the kinetic energy of a planetesimal about 1200km in radius, arriving at the Earth's surface at 13km/sec, would be sufficient to eject two lunar masses to near-escape speeds. Although around half the original energy may well appear as kinetic energy of the ejecta (Gault, 1964), whether or not there is sufficient mass in the high speed tail to eject two lunar masses is unknown.[1] Assuming that a large enough collision occurs after the Earth's core had formed or was forming, the ejected material would be already depleted in iron, as in the fission theory. Advantages of collision over fission are: (1) an energy source is provided to raise the material off the Earth, and (2) the theory is not purely evolutionary, depending on a chance encounter so that it does not require prediction of similar satellites for Mars or other planets.

The material ejected into orbit forms a cloud of hot dust, rapidly depleted in volatiles. As shown by Soter (1971), the particles in such a swarm would interact and rapidly collapse into the equatorial plane, where a satellite could form. The evolution at this point resembles that postulated by Ringwood, except that an energy source is provided that does not necessarily apply to all planets.

Figure 9 shows a schematic view of the evolution of planetesimals in a toroidal volume around the Earth's orbit. The time-scale and particle ages follow merely from assumed reasonable conditions (a combination of Figs. 2, 3, and 5), but the qualitative aspects, the formation of many sizeable secondary bodies, and their gradual depletion by collision with earth (based on Fig. 8) or each other (based on Fig. 7) describe a probable history. A possible large collision resulting in formation of the Moon is also shown.

This model has an important philosophically satisfying aspect. There has always been difficulty in accounting for all properties of all satellite systems by a single evolutionary theory. Jupiter and Saturn have "miniature" solar systems with retrograde outriders. Uranus has its spin and satellites' angular momentum vectors radically altered. Earth is a "dual" planet with a relatively huge satellite. Mars has only two tiny moons. Venus and Mercury have none. This heterogeneity becomes

[1] A. G. W. Cameron (1974, private communication) has been studying essentially the same model and suggests a much larger body, comparable in size to Mars. Such a large body is not ruled out by our work. See Gault *et al.* (1963a) for estimates of the energy partition.

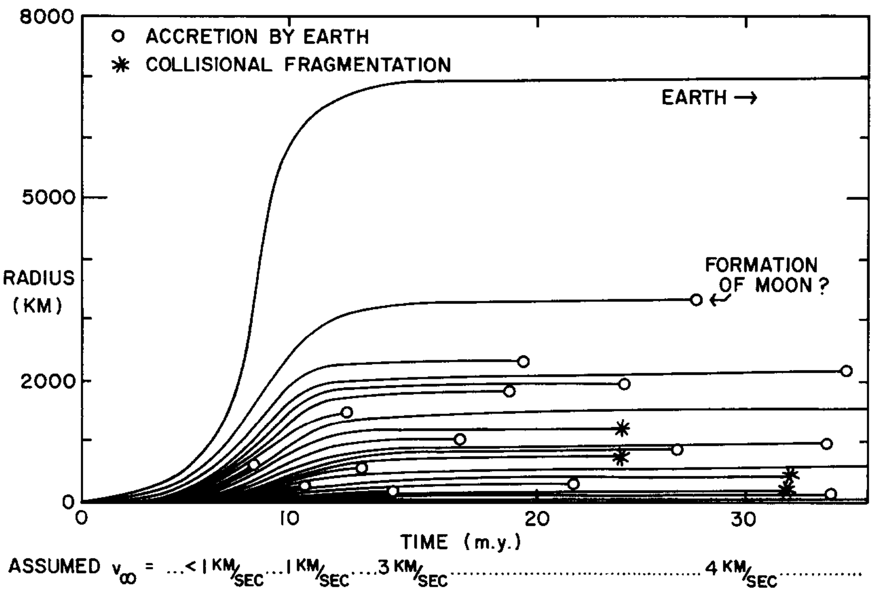

FIG. 9. Schematic history of planetesimal growth and related events near the Earth's orbit. Reality or timing of specific events is not claimed, but general character of growth of large planetesimals and their gradual depletion appears a necessary consequence of the theory. The nature of the most energetic collision between the Earth and the second-largest body may have been critical to the Moon's formation.

more satisfyingly accountable if it is viewed as the product of events involving statistics of small numbers. Does the second-largest planetesimal in each system hit the planet after 10^7 years or 10^8 years? Is it large or small? Does it hit the planet dead center? Retrograde? A glancing blow prograde? Or is it captured? Or is it destroyed by a planetesimal–planetesimal collision so that it has no appreciable effect on the planet other than to produce many small craters? Or does it hit a preexisting satellite of the planet, perhaps converting it to several small satellites? Only one of these kinds of fates can befall the second-largest planetesimal. And this fate, the product of small-number statistical chance encounters, may determine whether the planet acquires a tilted axis, a massive circumplanetary swarm of dust, a captured satellite, or perhaps loses a larger satellite, gaining small fragmentary satellites.

This model can thus account for the iron depletion, refractory enrichment, and volatile depletion of the Moon, and at the same time account for the Moon's uniqueness; the Moon may have originated by a process that was likely to happen to one out of nine planets.

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DISCUSSION

ALAN HARRIS: Safronov (1972) has studied the growth of V_∞ as planetesimals accrete and finds the expression

$$V_\infty{}^2 = GM/\theta R,$$

where M and R are the mass and radius of the largest body in the zone. The dimensionless parameter θ has a value of 3 to 5 over a wide range of size distributions. Using this value for V_∞, he proceeds to analytically study the terminal size distributions of planetesimals, and obtains the result that the radius of the second largest body is

$$r_2 = \frac{r_1}{1 + 2\theta} \approx \frac{r_1}{9},$$

thus implying $m_2/m_1 \approx 10^{-3}$. It would be very interesting to compare the results of a numerical study such as Hartmann's, using the above expression for V_∞, with Safronov's analytical approximations.

The differences between Safronov's result ($m_2 \approx 10^{-3} m_1$) and Hartmann's result ($m_2 \gg 10^{-3} m_1$) can be qualitatively understood as follows: (a) Safronov's expression for V_∞ is generally less than Hartmann's, hence the largest body obtains more help from gravitational focusing; and (b) even in the very beginning, V_∞ scales along with the size of the largest body such that it *always* benefits from gravitational focusing by a factor of $(1 + 2\theta)$, not just after reaching some critical size.

HARTMANN AND DAVIS: Our results are not entirely inconsistent with the Safronov result, $r_2 = 0.09\, r_1$, which you mention. For instance our run in Fig. 4 shows $r_2 \simeq 0.05\, r_1$ while our run in Fig. 5 gives $r_2 \simeq 0.2 r_1$. It would be proper to say our results bracket Safronov's though we were especially interested in illustrating conditions that could produce large r_2/r_1 ratios, and we found these to be reasonable conditions.

We have been influenced by some of the early Soviet accretion theories, published in the 50's and 60's, but have not yet been able to review Safronov's useful book in detail. We believe work in these areas will be extremely fruitful. Our V_∞ model here is rather arbitrary but it does increase monotonically, as does Safronov's. We have proposed research programs to study especially the evolution of V_∞, as it is critical to the growth process. It is not obvious to us that V_∞ should always be near the critical value that puts r at the critical transition radius between geometric and gravitational accretion.