

**El Niño Chaos: Overlapping of Resonances  
Between the Seasonal Cycle and the  
Pacific Ocean-Atmosphere Oscillator**

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# El Niño Chaos: Overlapping of Resonances Between the Seasonal Cycle and the Pacific Ocean-Atmosphere Oscillator

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The El Niño–Southern Oscillation (ENSO) cycle is modeled as a low-order chaotic process driven by the seasonal cycle. A simple model suggests that the equatorial Pacific ocean-atmosphere oscillator can go into nonlinear resonance with the seasonal cycle and that with strong enough coupling between the ocean and the atmosphere, the system may become chaotic as a result of irregular jumping of the ocean-atmosphere system among different nonlinear resonances. An analysis of a time series from an ENSO prediction model is consistent with the low-order chaos mechanism.

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**E**l Niño (roughly defined as the warming of the east equatorial water of the Pacific Ocean about every 3 to 6 years) and the accompanying Southern Oscillation signal in the atmosphere dramatically affect the Earth's climate on a global scale. The onset, termination, and cyclic nature of the ENSO events seem to be well explained by the linear equatorial wave dynamics and the delay oscillator idea (1–5). However, their irregular occurrence and partial locking to the regular seasonal cycle [El Niño events usually peak in the northern winter (6)] have been difficult to explain.

Here we use a simple delay equation model, including idealized seasonal forcing, to evaluate whether ENSO might be a low-order chaotic process driven by the seasonal cycle. We then analyze the chaotic behavior of the ENSO model of Cane and Zebiak [(7), hereafter CZ]. A companion paper (8) describes another test of this theory that uses a model that is fuller than our simple delay model and yet simpler (and therefore more accessible) than the CZ model from which we analyze a time series here.

The delay oscillator mechanism can be described as follows: A positive sea-surface temperature (SST) perturbation along the

eastern equatorial Pacific weakens the easterly winds above the equator [the Bjerknes hypothesis (9)]. The change in the winds excites a downwelling (deepening) wave in the thermocline that travels eastward to the South American coast as equatorial Kelvin waves and an upwelling signal that travels westward as equatorial Rossby waves. The downwelling Kelvin waves enhance the warming off the coast of South America—the El Niño event has begun. Subsequently, the westward-traveling upwelling Rossby waves are reflected from the western boundary of the Pacific Ocean as upwelling Kelvin waves, which travel eastward to counter the downwelling Kelvin waves, ultimately terminating the El Niño event.

We used a simple heuristic model of this accepted delay mechanism, including a Kelvin wave, one Rossby wave mode, and a dynamic link from mid-Pacific wind stress anomalies to these equatorial wave modes. To these we added a phenomenological seasonal forcing term representing the effects of the numerous seasonally varying features of the equatorial Pacific ocean and atmosphere, such as wind amplitude and SST variations (10). Apart from the seasonal forcing term used here, our model is basically a continuous version of the discrete delay equation given in (5) and is similar to those of (1–4). The equation is written for  $h(t)$ , the thermocline depth deviations from seasonal depth values at the eastern boundary

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$$\frac{dh(t)}{dt} = aA(h\{t - [L/(2C_K)]\}) - bA(h\{t - [L/C_K + L/(2C_R)]\}) + c \cos(\omega_a t) \quad (1)$$

where  $t$  is time,  $L$  is the basin width, and  $\omega_a$  is the annual frequency of the idealized seasonal forcing. The first term on the right represents a wind-forced Kelvin mode that travels at a speed  $C_K$ , and thus takes a time  $L/(2C_K)$  to reach the eastern boundary from the middle of the basin. The second term is due to the westward-traveling Rossby wave of speed  $C_R$  excited by the wind at time  $t - [L/C_K + L/(2C_R)]$  and reflected as a Kelvin wave.

The function  $A(h)$  relates wind stress to SST and SST to thermocline depth. The nonlinear form of  $A(h)$  [as in (5), equation 9] reflects the non-uniform stratification of the ocean. The slope of  $A(h)$  at  $h = 0$ , set by the parameter  $\kappa$ , is a measure of the strength of the coupling between ocean and atmosphere (11).

As the nonlinearity (as measured by  $\kappa$ ) is increased, the model follows the universal quasi-periodicity route to chaos (Fig. 1) (12). For small values of  $\kappa$ , the model time series is perfectly periodic at the annual period of the forcing (Fig. 1A, upper panel). The phase-space trajectory degenerates to a single point because of the sampling at

the annual frequency (Fig. 1A, middle panel), and the power spectrum shows a single peak at the annual frequency (Fig. 1A, lower panel).

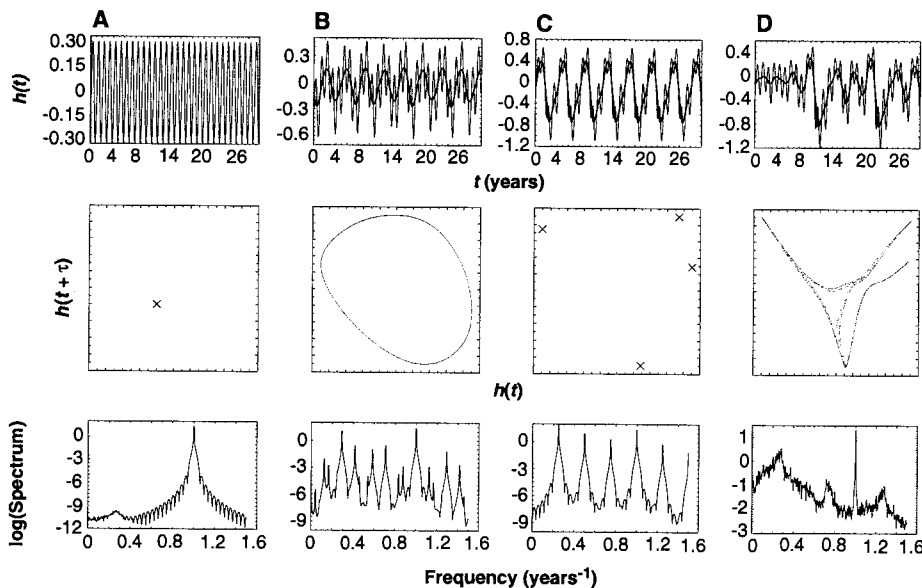
As  $\kappa$  increases (Fig. 1B), a second frequency ( $\omega_n$ ) arises, that of the natural oscillator of the Pacific ocean-atmosphere system (1-5). This second frequency is, in general, incommensurate with the annual frequency; the superposition of two incommensurate frequencies creates a quasi-periodic time series. The resulting oscillations are irregular but not chaotic (Fig. 1B, upper panel); the phase-space reconstruction is a simple closed loop (Fig. 1B, middle panel) (12), and the power spectrum shows two dominating frequencies ( $\omega_n = 0.29$  and the annual  $\omega_a = 1 \text{ year}^{-1}$ ) with many subharmonics (Fig. 1B, lower panel). These irregular oscillations are not locked to the seasonal cycle.

For even stronger nonlinearity (Fig. 1C), the system becomes mode-locked: The frequency of the nonlinear delay oscillator changes slightly to a simple rational multiple of the driving annual frequency:  $\omega_n = \omega_a P/Q$ , with  $P$  and  $Q$  integers (in Fig. 1C,  $\omega_n = \omega_a/4$ ). This model state corresponds to a nonlinear resonance between the driving annual frequency ( $\omega_a$ ) and the natural oscillator frequency ( $\omega_n$ ). The time series is periodic, and the phase-space diagram is a set of points whose number depends on the

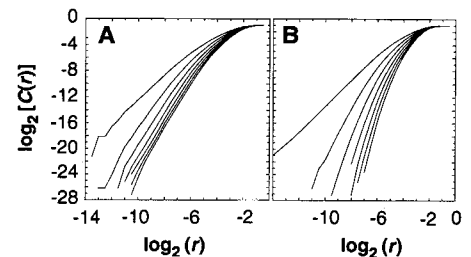
values of  $P$  and  $Q$ . The parameter regimes corresponding to the mode-locked solutions are also known as "Arnold tongues" (12).

Finally, for sufficiently large nonlinearity, the system becomes chaotic (Fig. 1D). The time series is irregular, the phase-space reconstruction is of a strange attractor, and the power spectrum is broad and not made of distinct peaks as before. For small nonlinearity, the model solution may be either quasi-periodic or mode-locked to a single nonlinear resonance defined by a single ratio  $P/Q$ . For larger nonlinearity, two or more mode-locked solutions (that is, solutions with different ratios  $P_i/Q_i$ ) may coexist; the nonlinear resonances are said to overlap in this case. The chaotic behavior is caused by the irregular jumping of the system among the different possible resonances (12). The quasi-periodicity route to chaos is a two-parameter route. One parameter governs the transition from quasi-periodicity to chaos as shown above; the second parameter governs the width of the mode-locked regimes, as investigated by Jin *et al.* (8).

We suggest that ENSO characteristics may be consistent with low-order chaotic behavior of the type described (Fig. 1D), which would account for ENSO's irregularity and its locking to the seasonal cycle. Each mode-locked solution (Fig. 1C) is in perfect phase correlation with the driving annual frequency. But even in the chaotic regime, as the system jumps irregularly among resonances, it remains partially locked to the



**Fig. 1.** The transition to chaos of the delay model. The data for these plots were obtained by running Eq. 1 for 1024 years to remove all transients and then for 1024 more years to obtain the data for the analyses. Shown are a time series of the model results (upper panels, thin lines), together with a 12-month running average of these results (upper panels, thick lines); a phase-space diagram (middle panels); and a logarithm of the power spectrum (lower panels) for the model time series. The reconstructed phase-space diagram is obtained by plotting  $h(t)$  versus  $h(t + \tau)$ , where  $\tau$  is the delay time (19) (not to be confused with the delay times of the delay oscillator), chosen here to be 1 year. The time series for  $h(t)$  is subsampled at the frequency of the external forcing, that is, at 1-year intervals (12). The four cases shown are (A)  $\kappa = 0.9$ , (B)  $\kappa = 1.2$ , (C)  $\kappa = 1.5$ , and (D)  $\kappa = 2.0$  and  $b_+ = 1.5$ . The few points contained in the phase-space diagram in (A) and (C) are denoted by "x."



**Fig. 2.** Correlation dimension estimates for (A) the CZ model (18) and (B) the linear Markov model. Shown is a  $\log_2$ - $\log_2$  plot of the Grassberger-Procaccia (15) correlation function as a function of the distance between trajectory points in reconstructed phase space. Each curve is for a different embedding dimension  $d_E$ , with the uppermost for  $d_E = 2$  and the rest for  $d_E$  increasing in steps of 2. The dimension of the attractor that produced the time series is given by the slope of the linear segment in these curves. The curves calculated for the CZ model contain a linear segment whose slope converges to a constant value as the embedding dimension increases. The dimension calculated from these curves is about 3.5. The slopes for the Markov model time series (B) do not seem to converge as quickly as those for the CZ model time series for increasing embedding dimension, which suggests instead a higher dimension, as is characteristic of randomly driven linear systems.

driving seasonal frequency: The maxima of the 12-month running average of the model time series [representing the ENSO warm events (Fig. 1D, upper panel)] occur every 2 to 5 years and always within the same 4 months of the calendar year—precisely the ENSO characteristic missing in the existing simplified delay-oscillator ENSO models. Earlier examinations of ENSO as a low-order chaotic system either have used simplified models lacking the essential equatorial wave dynamics (although they discussed the locking to the seasonal cycle) (13) or did not fully realize the importance of mode locking to the seasonal cycle and the mechanism of resonance overlapping (5).

Is the irregularity of ENSO indeed due to low-order chaos and not to random forcing (2)? The instrumental record of the real ENSO data, which extends over slightly more than 100 years, is too short to identify chaos in an observed time series. Instead, we analyzed the results of the CZ ENSO model (7), which has been used to predict several ENSO events (14).

The diagnostic tool we used to identify chaotic model behavior was the calculation of the phase-space correlation dimension (15) from monthly averaged East Pacific SST from a 1024-year run. The correlation dimension for this run is about  $d \approx 3.5$  (Fig. 2A), which suggests a chaotic dynamic system with a small [ $\leq (2d + 1)$ ] number of degrees of freedom.

The correlation dimension calculations are prone to various artifacts (16), and in order to reduce this possibility we used a control time series of surrogate data (17) with the same characteristics (number of points, power spectrum) as the CZ model time series. We chose a time series from a linear Markov model built from the CZ model and driven by random forcing (18). The dimension estimate for the Markov model (Fig. 2B) indicates that this time series is random and distinguishable from the low-order dimension found with the CZ model. This result is consistent with the suggestion that the irregularity of ENSO events (at least in the CZ model) is not due to random noise (such as ocean weather phenomena present in the CZ model).

We suggest that the natural oscillator of the equatorial Pacific ocean-atmosphere system can enter into nonlinear resonance with the seasonal cycle at several periods of the oscillator (mostly 2 to 5 years). The coexistence of these resonances results in chaotic behavior that is due to the jumping of the system among the different resonances. This is a feature of the quasi-periodicity route to chaos (12).

Much additional work is needed to further examine the relevance of these ideas to the observed ENSO characteristics and to clarify the spatial and temporal mechanisms

of the seasonal forcing of Pacific interannual variability. If this theory can be validated, the ENSO cycle might be established as an example of low-order chaos in a highly complex physical system.

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