ES128: Homework 2
Due in class on Wednesday, 03 March 2010

Problem 1

Show that the weak form of

$$\frac{d}{dx}(AE\frac{du}{dx}) + 2x = 0 \qquad \text{on } 1 < x < 3,$$

$$\sigma(1) = \left(E\frac{du}{dx}\right)_{x=1} = 0.1,$$

$$u(3) = 0.001$$

is given by

$$\int_1^3 \frac{dw}{dx} AE \frac{du}{dx} dx = -0.1(wA)_{x=1} + \int_1^3 2xw dx \qquad \forall w \text{ with } w(3) = 0.$$

Problem 2

Consider the (steel) bar in Figure 1. The bar has a uniform thickness t=1 cm, Young's modulus E=200×10^9 Pa, and weight density ρ=$7\times10^3\ kg/m^3$. In addition to its self-weight, the plate is subjected to a point load P=100N at its midpoint.

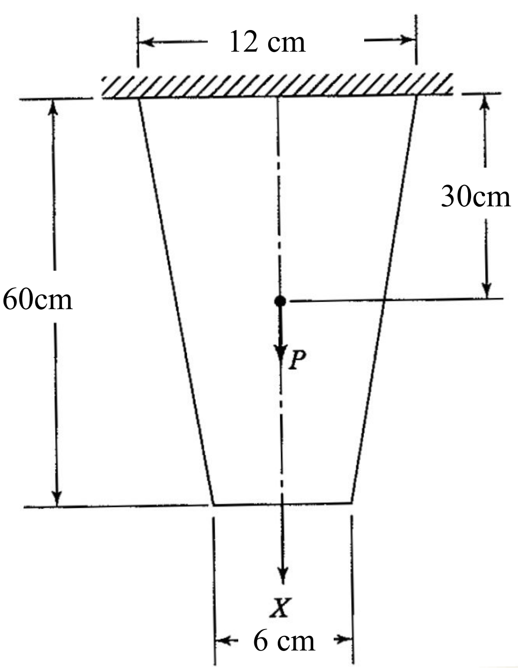

Figure 1

(a) Model the bar with two finite elements.
(b) Write down expressions for the element stiffness matrices and element body force vectors.
(c) Assemble the structural stiffness matrix K and global load vector F.
(d) Solve for the global displacement vector d.
(e) Evaluate the stresses in each element.
(f) Determine the reaction force at the support.

Problem 3

Consider the mesh shown in Figure 2. The model consists of two linear displacement constant strain elements. The cross-sectional area is *A*=1, Young's modulus is *E*; both are constant. A body force *b(x)=cx* is applied.

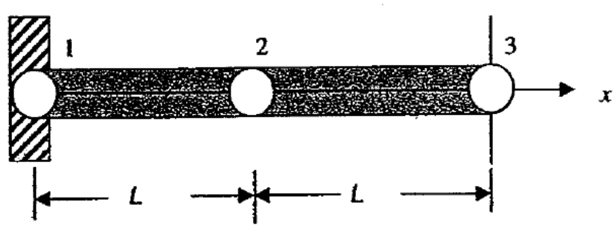

Figure 2

(a) Solve and plot *u(x)* and $\varepsilon(x)$ for the FEM solution.
(b) Compare (by plotting) the finite element solution against the exact solution for the equation

$$E\frac{d^2u}{dx^2} = -b(x) = -cx.$$

(c) Solve the above problem using a single quadratic displacement element.
(d) Compare the accuracy of stress and displacement at the right end with that of two linear displacement elements.
(e) Check whether the equilibrium equation and traction boundary condition are satisfied for the two meshes.