ES128: Homework 3
Due in class on Wednesday, 17 March 2010

Problem 1

Derive the equilibrium equations and the relations between strains ($\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{zz}, \varepsilon_{r\theta}, \varepsilon_{rz}, \varepsilon_{\theta z}$) and displacements in cylindrical coordinates with $x_1 = r\cos\theta$, $x_2 = r\sin\theta$, and $x_3 = z$.

Problem 2

Elastic bulk modulus B is defined as the stiffness under hydrostatic pressure, namely,

$$p = -B\frac{\Delta V}{V}$$

Here $\Delta V / V$ is the volumetric strain caused by the hydrostatic pressure p.

(a) Show that the volumetric strain $\Delta V / V$ relates to the axial strains as

$$\frac{\Delta V}{V} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}.$$

(b) Express the bulk modulus B in terms of Young's modulus and Poisson's ratio.

(c) What is the value of Poisson's ratio when the material is incompressible? Interpret your result under uniaxial stress state.

(d) Show that Poisson's ratio must be smaller than ½. What would happen if Poisson's ratio were greater than ½?

Problem 3

Stress concentration at geometric discontinuities is the most important practical result in elasticity theory. For example, for a small circular hole in a large plate under uniaxial stress S, the elasticity solution gives the hoop stress around the hole:

$$\sigma_{\theta\theta} = S(1 - 2\cos 2\theta)$$

Here the polar angle θ is measured from the loading direction. The problem is solved in many elasticity textbooks.

a) Under uniaxial tension, indicate the highest tensile stress around the hole.

b) Under uniaxial compression, indicate the highest tensile stress around the hole.

c) Calculate the stress concentration at the hole when the plate is under a pure shear stress. Use the above solution and linear superposition.

Problem 4

An elastic layer is sandwiched between two perfectly rigid plates, to which it is bonded. The layer is compressed between the plates, the direct stress being σ_{33}. Supposing that the attachment to the plates prevents lateral strain ε_{11}, ε_{22} completely, find the apparent Young's modulus (that is $\sigma_{33}/\varepsilon_{33}$) in terns of E and ν. Show that it is many times E if the material of the layer has a Poisson's ratio only slightly less than 0.5, e.g., rubber.